Geometry of Supermagnets

Volker Schomerus

Edinburgh, Jan 2011

**based on work with C. Candu,
T. Creutzig, V. Mitev, T. Quella,
H. Saleur; Y. Aisaka,
N. Berkovits, T. Brown**

Motivation: AdS/CFT

Supersymmetric Statistical System

(String-) Geometry

4-dim SUSY gauge theory (N=4 SYM)

Maldacena

Asymtotically AdS_5 geometry (AdS_5 x S^5)

$N_c \rightarrow \infty$

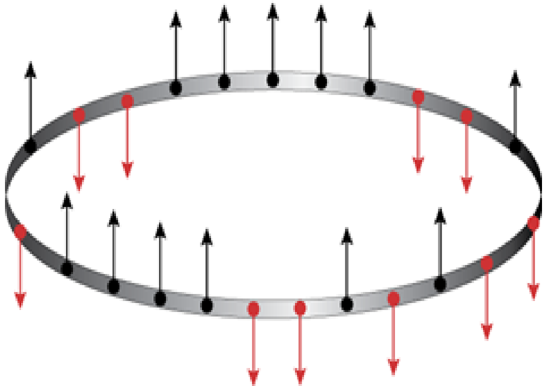
re-packaging

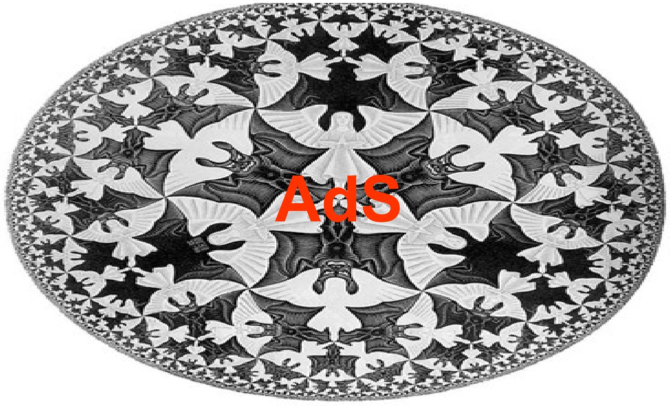
'tHooft, Polyakov

Supermagnet

?

~ twistor string $\leftrightarrow$ $C^{3|4}$





Challenge of Gauge/String duality

2D critical systems w continuous spectrum, worldsheet SUSY, internal supersymmetry

EOM of String Theory
in covariant formulation

Liouville mode of ST
e.g. AdS backgrounds

Sufficient #(couplings)
e.g. RR-fluxes

String description of
SUSY Gauge Theories

Challenge: New class of non-unitary, non-rational, superconformal theories IHP workshop Sep-Dec 2011

Advanced Conformal Field Theory

Example: OPS(2S+2|2S) GN model

Experience: Worldsheet SUSY & continuous spectra [Liouville]

Very little with internal SUSY ← disordered systems SUSY trick [Efetov]

OSP(2S+2|2S) covariant version of masslessThirring:

Gross-Neveu

2S+2 real fermions

S βγ-systems c=-1

$h_\psi = h_\beta = h_\gamma = 1/2$

$$\mathcal{S}^{\mathrm{GN}} \sim \int d^2z \left[\sum_i \psi_i \bar{\partial} \psi_i + \sum_a \beta_a \bar{\partial} \gamma_a\right] + cc$$

c=1 CFT with affine osp(2S+2|2S) ; k=1

$$+g^2 \int d^2z \left[\sum_i \psi_i \bar{\psi}_i + \sum_a (\gamma_a \bar{\beta}_a - \beta_a \bar{\gamma}_a)\right]^2 \quad \sim J_\mu J^\mu$$

↔ fermionic sector of NSR superstring in curved background

1-parameter family of interacting CFTs with c=1

no KM sym!

Emergent Geometry

Massless Thirring model: O(2) statistical sys.

real fermions

$$\mathcal{S}^{\mathrm{Th}}_{m=0} \sim \int d^2 z \sum_{i=1}^{2} \left[\psi_i \bar{\partial} \psi_i + cc + g^2 (\psi_1 \bar{\psi}_1 + \psi_2 \bar{\psi}_2)^2 \right]$$

i=1,2

Discrete version is XXZ spin chain **[Luther 1976]**

Jordan-Wigner transform

$$H_{XXZ} \sim \frac{1}{4} \sum_{j=1}^{L-1} \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j + \Delta \left(c_j^\dagger c_j - c_{j+1}^\dagger c_{j+1} \right)^2 \right)$$

Massless Thirring $\leftrightarrow$ compactified free Boson

[Coleman 1975], [Mandelstam 1975]

$R^2 = 1 + g^2$

Does not extend to O(N) models

$\leftrightarrow$ isolated WZW models: no separation of mass-less/ive modes

Main results and Plan

OSP(2S+2|2S) Gross-Neveu model with S>0

- discrete analysis: **OSP(2S+2|2S) XXZ $\leftrightarrow$ loop model**

  Numerics $\rightarrow$ harmonics of supersphere $S^{2S+1|2S}$ at $g = \infty$

- continuum theory: **Exact computation of P.F. $Z^g(q)$**

  OSP(2S+2|2S) GN $\leftrightarrow$ σ model on supersphere $S^{2S+1|2S}$

similar results exist for PSU(N|N) **[Candu,Mitev,Quella,VS,Saleur]**

Non-rational CFTs$^{c=0}$ with ws & internal SUSY

ws SUSY GN models [D'Adda,Luscher,Di Vecchia] ~ G/G models [Berkovits...]

II.1 Spin Chain: From O(2) to OSP

$$H_{XXZ} = -\frac{1}{2}\sum_{j=1}^{L-1}\left(\sigma_j^x \otimes \sigma_{j+1}^x + \sigma_j^y \otimes \sigma_{j+1}^y + \Delta\sigma_j^z \otimes \sigma_{j+1}^z\right)$$

acts on V_f^L **with** $V_f = \mathbb{C}^2$

Permutation P: **P** $e_a \otimes e_b = e_b \otimes e_a$ **P = X**

Projection E: **E** $e_a \otimes e_b = \delta_{ab} \sum e_c \otimes e_c$ **E =** 

I = | |

$$H_\Delta = -\frac{1+\Delta}{2}\sum_{j=1}^{L-1}(I + P_j) - \frac{1-\Delta}{2}\sum_{j=1}^{L-1} E_j$$

Universal expression
for all OSP(2S+2|2S)

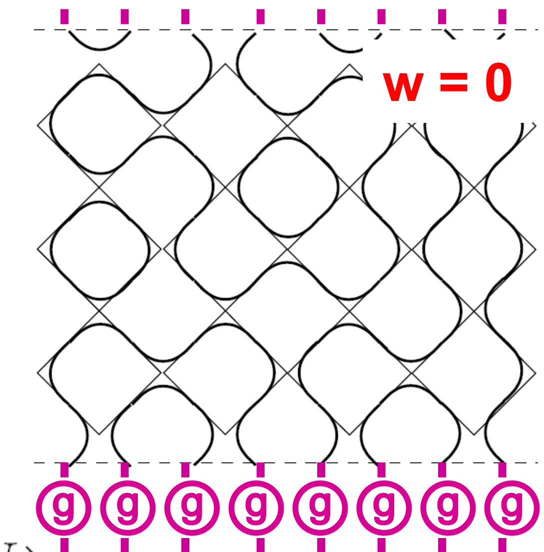
acts on V_F^L **with** $V_F = \mathbb{C}^{2S+2|2S}$

For S > 0 these spin chains are not integrable

II.2 Reformulation as loop model

Transfer matrix: $t(w) = I + E + wP$

$$T(w) = \prod_j t_{2j}(w) \prod_j t_{2j-1}(w)$$



Partition function: g ∈ OSP(2S+2|2S)

$$Z^w_{\mathbf{S}}(q;g) = \lim_{\binom{N\to\infty}{L\to\infty}} str\left(g^{\otimes L}\, T(w)^N\right)$$

$$N/L = \beta$$

$q = e^{-\beta}$

[Read,Saleur]

w ≠ 0

S = 0: orientation

Sum over intersecting loop patterns & super-colors

S=0 ↔ simple height model – discrete path integral for Φ

II.3 Some Numerical Results

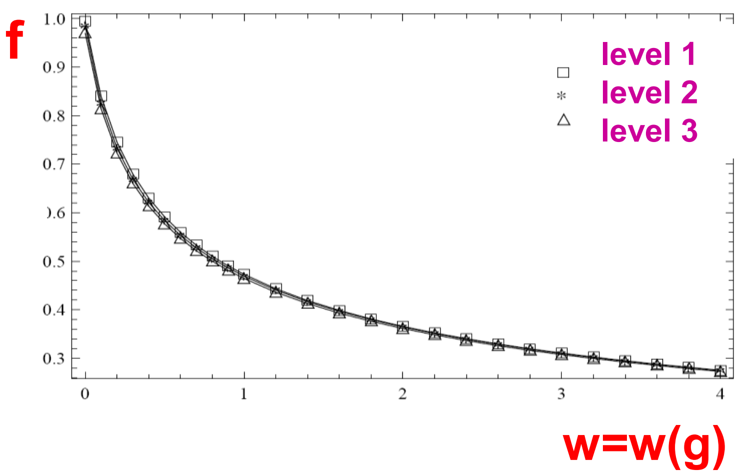
...with free boundary conditions **[Candu, Saleur]**

At large w: ∞ many states possess $\Delta \sim 0$

transform in trivial & irreps $\left[\frac{1}{2}, \frac{k-1}{2}, \frac{k-1}{2}\right]$ **of osp(4|2)**

dim = $4k^2+2$ k=1,2,3,....

Casimir evolution of conformal weights !



$f_\Phi(w) = \delta\Delta_\Phi / C_{\Lambda(\Phi)}$

$\delta\Delta_\Phi = \Delta_\Phi(w) - \Delta_\Phi(0)$

$f_\Phi = f$ is universal

III.1_1 Continuum analysis of GN

OSP(2S+2|2S) GN model$_{g = 0}$ with free boundary conditions

OSP(2S+2|2S) affine algebra at level k = 1

↔ gluing cond $J = \bar{J}$

$$Z_{S=0}^{g^2=0} = \sum_{m=-\infty}^{\infty} \frac{q^{\frac{m^2}{2}}}{\eta(q)} z^m = \frac{\theta_3(q, z^{1/2})}{\eta(q)}$$

III.1_2 Continuum analysis of GN

OSP(2S+2|2S) GN model$_{g=0}$ with free boundary conditions

OSP(2S+2|2S) affine algebra at level k = 1 ↔ gluing cond $J = \bar{J}$

$$Z^{g^2=0}_{S=1} = \frac{\theta_3(q^2,z_2)\theta_3(q^2,z_3)+\theta_2(q^2,z_2)\theta_2(q^2,z_3)}{\eta(q)\theta_4(q,z_1^{1/2})}$$

sum of two osp(4|2) characters at k=1

$$\sim q^{-\frac{1}{24}}\left(1+q^{1/2}\chi_F+q\chi_{ad}+\dots\right)$$

1 identity fld **6 flds (ψ,β,γ) with Δ = ½** **17 currents**

z_a ↔ parametrize elements g from the maximal torus is OSP(4|2)

III.2 Casimir evolution of Weights

Free Boson:

In boundary theory
bulk more involved

$$\Delta^g_\Phi = \Delta^0_\Phi + f(g)\, m^2_\Phi$$

at g=0   universal   U(1) charge

Prop.: Boundary weights of OSP(2S+2|2S) GN:

$$\Delta^g_\Phi = \Delta^0_\Phi + f_s(g) C^{(2)}_\Phi$$

quadratic Casimir

$f_s = f_0 \leftarrow$ cohomological reduction

Casimir evolution of the conformal weights Δ

[Bershadsky et al] [Quella,VS,Creutzig] [Candu, Saleur]

Ex: mult. (ψ,β,γ) $\Delta^g = \Delta^{g=0} + f(g)\, C_F = ½ + f(g)\, 1 \rightarrow 0$

fund rep: $C_F = 1$ $g \rightarrow \infty$

III.3 The Branching functions

From following decomposition of Z^g at g = 0

$$Z^{g=0}(q;z_i) = \sum_{\Lambda} \psi_\Lambda(q)\chi_\Lambda(z_1,z_2,z_3)$$

$\Lambda = [j_1, j_2, j_3]$ for osp(4|2)

characters

→ Branching functions

$$\psi_{[j_i]}(q) = \frac{1}{\eta(q)\phi(q)^3}\sum_{n,m=0}^{\infty}(-1)^{m+n}q^{\frac{m}{2}(m+4j_1+2n+1)+\frac{n}{2}+j_1}$$

$$\times\left(q^{(j_2-\frac{n}{2})^2}-q^{(j_2+\frac{n}{2}+1)^2}\right)\left(q^{(j_3-\frac{n}{2})^2}-q^{(j_3+\frac{n}{2}+1)^2}\right)$$

replace $\psi_m \sim q^{m^2/2}/\eta$, $\chi_m \sim z^m$ for massless Thirring

III.4 Spectrum of OSP(4|2) GN model

[Candu,Mitev,Quella,VS,Saleur]

Value of Quadratic Casimir in representation of osp(4|2)

$$C^{(2)}[j_1, j_2, j_3] = -4j_1(j_1 - 1) + 2j_2(j_2 + 1) + 2j_3(j_3 + 1)$$

can be positive and negative

- All Δ^g are bounded from below $\Delta^g > 0$

w=w(g)

- Provides explicit formula for $Z^w(q,z)$, S=1

III.5 The Supersphere σ-model

$$S^{2S+1|2S} = \{C := \sum_{i=1}^{2S+2} x_i^2 + 2\sum_{a=1}^{S} \eta_{2a-1}\eta_{2a} = 1\}$$

Family of CFTs with continuously varying exp.

parameter R

$$X = (x, \eta)$$

+ constraint

$$\mathcal{S}_R \sim R^2 \int d^2 z \partial X_a \bar{\partial} X^a \qquad C(X_a) = 1$$

cp. PCM on S^3 → massive flow

Solving constraints → non-linear action:

$$\mathcal{S}_R \sim R^2 \int d^2 z (1 - 2\eta_1\eta_2)\left(\partial\varphi_1\bar{\partial}\varphi_1 + \cos^2\varphi_1\,\partial\varphi_2\bar{\partial}\varphi_2 + \sin^2\varphi_1\,\partial\varphi_3\bar{\partial}\varphi_3\right) + \ldots$$

III.6$_1$ From GN to Supersphere

For massless Thirring model (S=0) we find

$$Z_{S=0}^{g^2=\infty} = \sum_m \frac{z^m}{\eta(q)} = q^{-\frac{1}{24}} \overbrace{\phi(q) \lim_{t\to 1}(1-t^2)}^{\text{implements } X_1^2 + X_2^2 = 1}$$

Euler function

$q^0 = t$

$$\prod_{n=0}^{\infty} \frac{1}{(1-z^{1/2}q^n)(1-z^{-1/2}q^n)}$$

generated by modes of X_1 X_2

Zero modes counted by $\sum z^m \leftrightarrow \exp(im\varphi)$

III.6_2 From GN to Supersphere

For OSP(4|2) Gross Neveu model we find

$$Z_{S=1}^{g^2=\infty} = q^{-\frac{1}{24}}\overbrace{\phi(q)\lim_{t\to 1}(1-t^2)}^{\text{implements Ss constraint}} \quad \text{x}$$

generated by modes of η_1 η_2

$$\text{x} \prod_{n=0}^{\infty} \frac{(1+z_1^{1/2}q^n)(1+z_1^{-1/2}q^n)}{(1-z_2^{1/2}z_3^{1/2}q^n)\cdots(1-z_2^{-1/2}z_3^{-1/2}q^n)}$$

generated by modes of X_1 X_4

Zero modes reproduce harmonics of supersphere $S^{3|2}$

Supermagnets for N = 4 SYM ?

Recall: **We need superconformal 2D CFTs (c=0) w. continuous spectrum and internal supersymmetry**

Candidates for a dual of N = 4 $\mathrm{SYM}_{\lambda=0}$ from N = 1 sc models on coset superspaces G/H

G = U(2,2|4); many choices for H

for appropriate choices of H → N = 2 sc symmetry **possess c = 0**

~ partially gauge fixed twisted G/G models

e.g. H = U(2,2) x U(4) [Berkovits,Vafa]

Note: **G/H does not look like AdS_5 x S^5 !** **where is AdS ?**

Fermionic sector & GN model

N=1 sc G/H models possess following form:

e.g. G=U(4|4); H=U(1)xU(1,2|4):

(X,η) **(ψ,γ,β)**

$$S \;=\; \frac{1}{2g_\sigma^2}\int d^2z\,\left[(\bar{D}^\mu Z_\alpha)^* D_\mu Z_\alpha\right.$$

$g^2 \sim 1/R^2$

GN-like sector

U(N) version [D'Adda,Luscher,DiVecchia], U(4|4) version [Witten]

<u>Summary</u>: Geometry may grow from fermionic sector of N = 1 super-conformal coset models.

Outlook

- Identify the dual of weakly coupled N=4 SYM

  in progress **w. Y.Aisaka,N.Berkovits,T.Brown,A.Michaelov,V.Mitev**

- Explore its moduli space (marginal couplings)
- Extend analysis of OSP(2S+2|2S) GN model by including ws SUSY **↔ OSP version of 19 vertex model**
- PSU(N|N) cases: For $CP^{N-1|N}$ similar results..

  No GN-like continuum description known yet **CY ↔ Gepner**

- Scan WZW super-coset models ↔ σ-models

  quantum integrable systems meet string geometry

Conclusions

The present work is a first step towards finding CFT-s describing sigma models. I think that it will require a work of many people to complete this task. I also believe that if we want to understand fundamental physics, there is no way avoiding it. The nature of space- time at large curvature is determined by gauge /strings correspondence and it , to the large extent, boils down to the sigma models, similar to the ones we discussed. It is possible that when better understood these ideas will influence our picture of the early universe.

from Polyakov, *Supermagnets and Sigma models*, hep-th/0512310