Bosonic Excitations in Holographic Quantum Liquids

Richard Davison

Rudolf Peierls Centre for Theoretical Physics, Oxford

In collaboration with N. Kaplis, A. Starinets; arXiv:1109.6343 and 1110.xxxx

Edinburgh Mathematical Physics Group Seminar

October 19th 2011

AdS/CFT and AdS/CMT

AdS/CFT tells us that

String theory configuration
e.g. N $\rightarrow \infty$ D3 branes

Classical theory of gravity
e.g. IIB SUGRA on $AdS_5 x S^5$

=

Strongly-coupled field theory
e.g. $\mathcal{N}$=4 SU(N) SYM

Can compute strongly-interacting field theory properties from gravity.

What are the low energy properties of these field theory states with a large density of matter ('holographic quantum liquids')?

Are they anything like those of real materials?

Outline

1. Landau's Theory of Fermi Liquids

2. Holographic Quantum Liquids

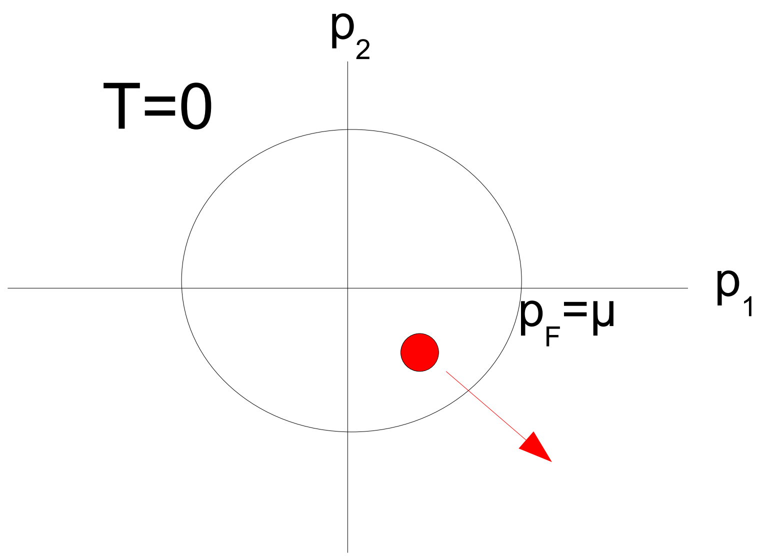
3. The D3/D7 Theory

4. The RN-AdS$_4$ Theory

5. Conclusions

Landau's Theory of Fermi Liquids

For non-interacting fermions, the ground state is a filled Fermi sphere.



Excited states are generated by exciting fermions as shown.

Landau's theory: when interactions are turned on, the ground state remains a filled Fermi sphere of fermionic quasiparticles.

These quasiparticles now interact over short distances and so excitations are more complicated.

The theory is valid provided that
- $T \ll \mu$: the Fermi sphere is a good description of the ground state.
- $\omega, q \ll \mu$: we only excite quasiparticles near its surface.

Bosonic Excitations

If the quasiparticle interaction is repulsive, there is a collective mode of the liquid corresponding to oscillations of the Fermi surface.

This is a density wave that propagates at zero temperature - it is called 'zero sound'. We can characterise it by its dispersion relation $\omega(T,q) = \bar{\omega}(T,q) - i\Gamma(T,q)$

decay rate

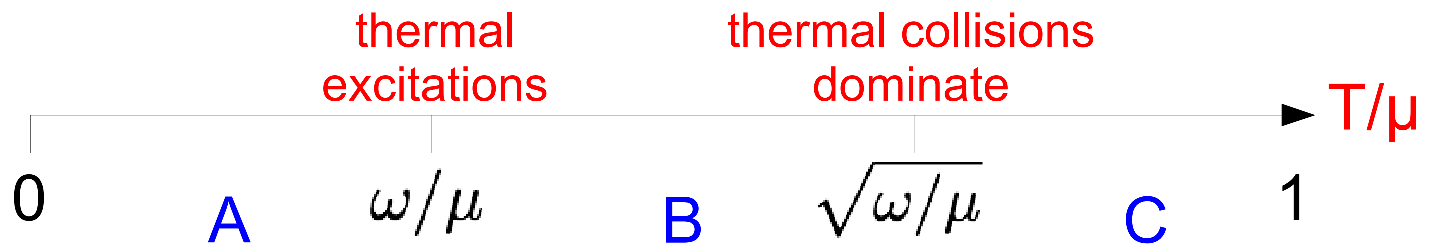
At non-zero temperatures T<<μ , there are two important scales:

- thermal excitations are important when $\omega \sim T$
- thermal collisions dominate when $\omega \sim T^2/\mu$

thermal collision frequency

The Three Regimes of Sound

The decay rate of sound is different in each of the three regimes



A: the collisionless quantum regime

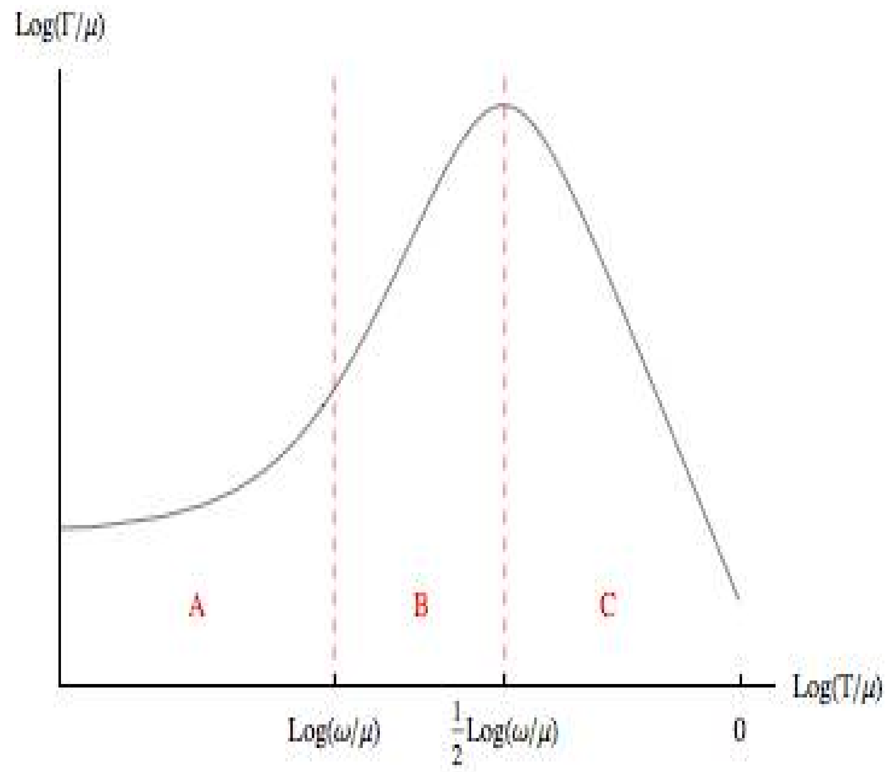
$$\Gamma(T,q) = \Gamma(0,q) \propto q^2/\mu$$

B: the collisionless thermal regime

$$\Gamma(T,q) = \Gamma(0,q) + \alpha\, T^2/\mu$$

C: the hydrodynamic regime
crossover to hydrodynamic
sound and diffusion:

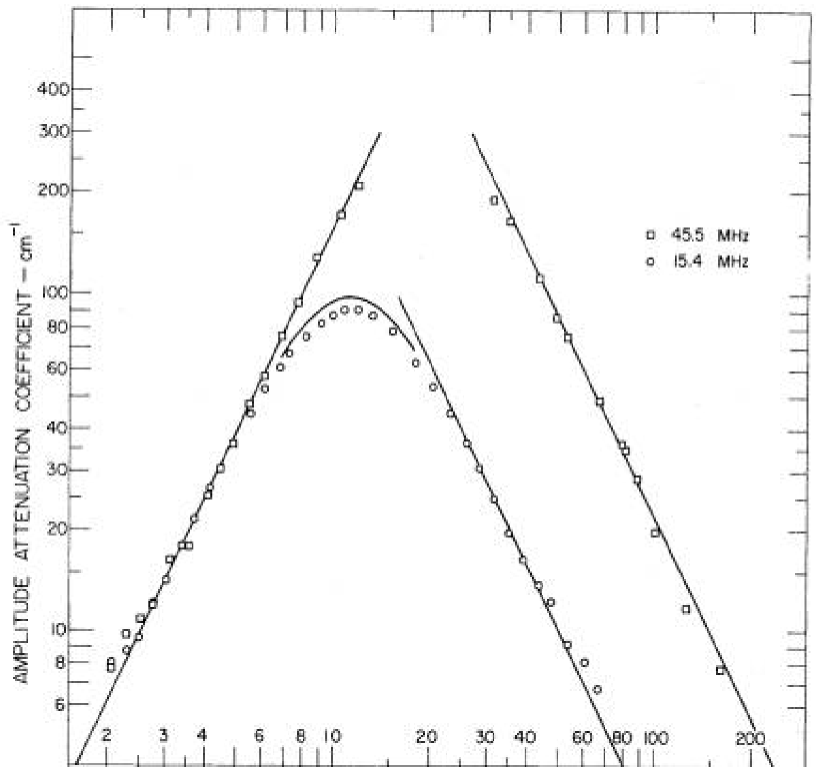
$$\Gamma(T,q) \propto \mu\,(\omega/T)^2$$



Liquid Helium-3

These properties were subsequently observed in liquid Helium-3.

Log(Γ/μ)



Log(T/μ)

Abel, Anderson and Wheatley
Phys. Rev. Lett. **17** (1966)

Its low energy properties are well described by Landau-Fermi liquid theory.

Holographic Quantum Liquids

How do holographic (i.e. strongly-coupled) theories with a large matter density ('holographic quantum liquids') behave?

From their gravitational duals, we can compute the following field theory properties:

- Thermodynamics and transport coefficients
- Fermionic excitations
- Bosonic excitations

How are these encoded?

Field Theory	Gravity
Operators $\{\mathcal{O}\}$	Fields $\{\varphi\}$
e.g. $T^{\mu\nu}$	$g_{\mu\nu}$
J^{μ} global symmetry	A_{μ}
Expectation Values $\langle\mathcal{O}\rangle$	$\varphi \xrightarrow[r\to\infty]{} J + \frac{\langle\mathcal{O}\rangle}{r} + \ldots$
Equilibrium State	Solution to EoMs

Retarded Green's Functions

The bosonic excitations are encoded in the retarded Green's functions of bosonic operators:

$$G^R(\omega, q) = -i \int d^{d-1}x\,dt\,e^{i(\omega t - q.x)}\theta(t)\langle[\mathcal{O}(t,x)\,\mathcal{O}(0,0)]\rangle = \left.\frac{\delta\langle\mathcal{O}\rangle}{\delta J}\right|_{J\to 0}$$

Poles $\omega(q)$ correspond to field theory excitations. Long-lived excitations are those with a small imaginary part.

To compute these from gravity, excite the fields around their background values and solve the equations of motion.

I will concentrate on the charge density operator. This should contain the most interesting excitations (sound and diffusion).

The D3/D7 Field Theory

How can we add fundamental matter in AdS/CFT?

Start with N_c D3-branes and N_f D7-branes. This couples N_f hypermultiplets (fermions and scalars) to $\mathcal{N}$=4 SYM. The resulting theory has $\mathcal{N}$=2 SUSY.

The $U(1) \subset U(N_f)$ global symmetry has an associated conserved charge J^t. If this charge has a non-zero VEV, we have a non-zero density of fermions + scalars.

The field theory is known exactly. But its gravitational dual is only known in the limit $N_f \ll N_c$ (and $N_c \rightarrow \infty$, $\lambda \rightarrow \infty$ as usual).

The dynamics of the energy-momentum tensor are identical to those of $\mathcal{N}$=4 SYM. But the dynamics of the charge are non-trivial.

The D3/D7 Gravitational Theory

The background solution of the gravitational theory has a non-trivial U(1) gauge field A_a in AdS$_5$xS^5:

$$S_{U(1)} \sim -\int d^8\xi \sqrt{-\det\left(g_{ab} + F_{ab}\right)}$$

Karch, Katz hep-th/0205036

This corresponds to a field theory state with non-zero density of fermions+scalars.

By exciting this gauge field, one can compute the charge density Green's function.

At T=0, there is only one long-lived excitation when ω,q<< μ. It has the dispersion relation

$$\omega = \pm \frac{q}{\sqrt{3}} - i\frac{q^2}{6\mu_0} + \ldots$$

“holographic zero sound”

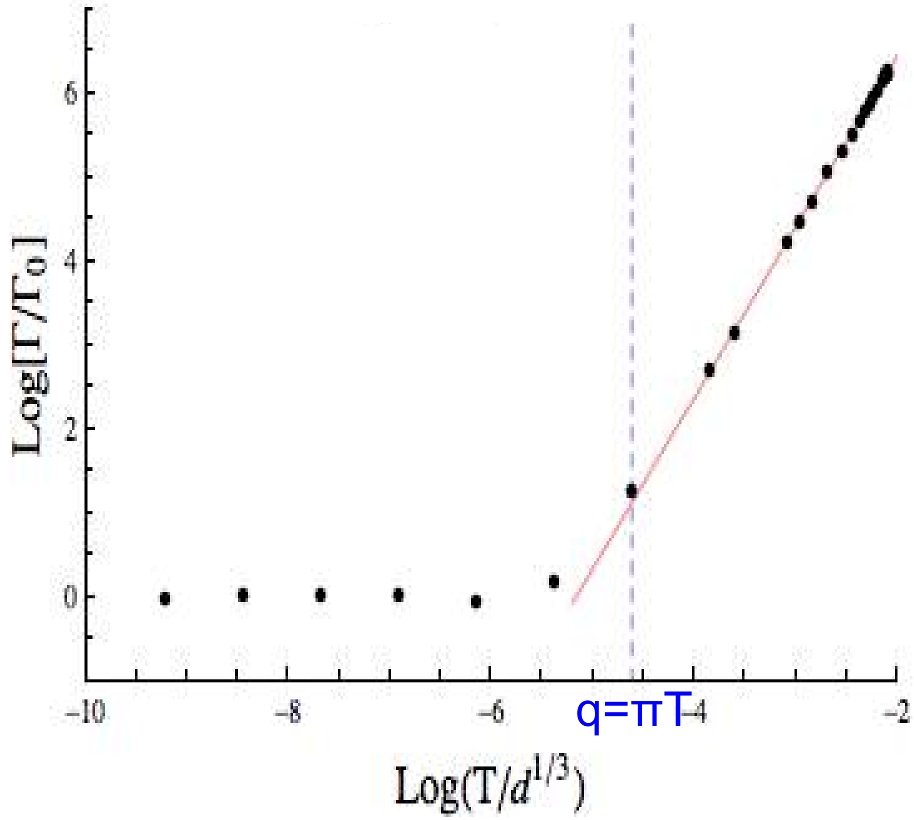
Karch, Son and Starinets
arXiv:0806.3796 [hep-th]

The D3/D7 Sound Mode

Does this behave anything like the zero sound of a Landau-Fermi liquid? Turn on the temperature T<<μ.

R.D. and A. Starinets
arXiv:1109.6343 [hep-th]

D3/D7 Theory



Landau's Theory

- A: quantum collisionless regime

$$\Gamma(T,q) = \Gamma(0,q) \propto q^2/\mu$$

- B: thermal collisionless regime

$$\Gamma(T,q) = \Gamma(0,q) + \alpha\, T^2/\mu$$

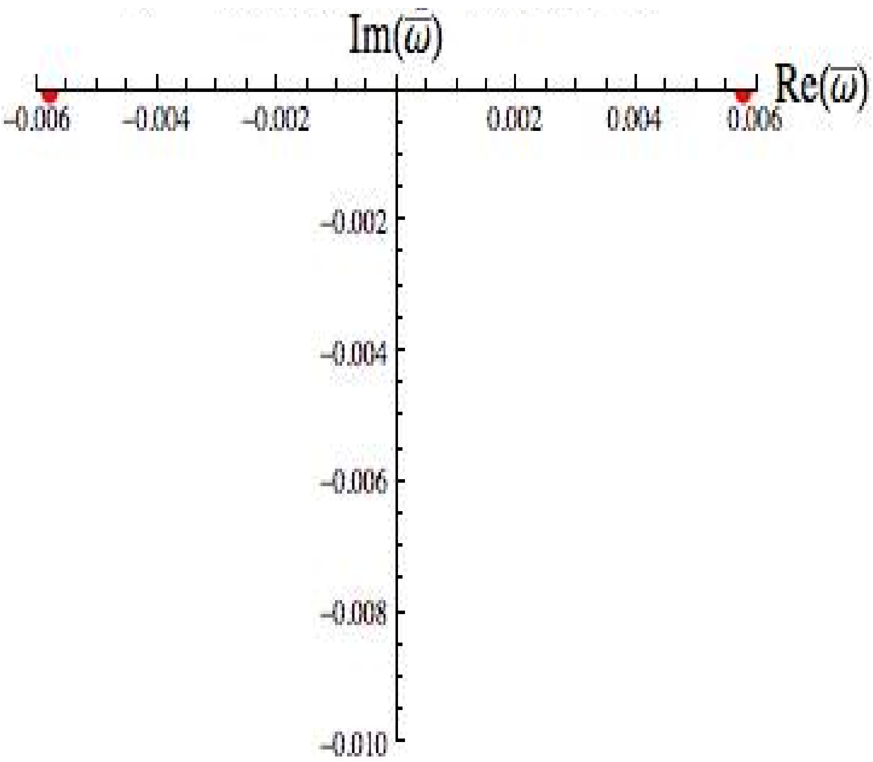
- C: hydrodynamic regime

  crossover to hydrodynamic sound and diffusion

$$\Gamma(T,q) \propto \mu\,(\omega/T)^2$$

The D3/D7 Theory Sound Mode

Is there a crossover from collisionless to hydrodynamic behaviour at even higher temperatures?



The zero sound modes become less stable and collide at a sufficiently high temperature.

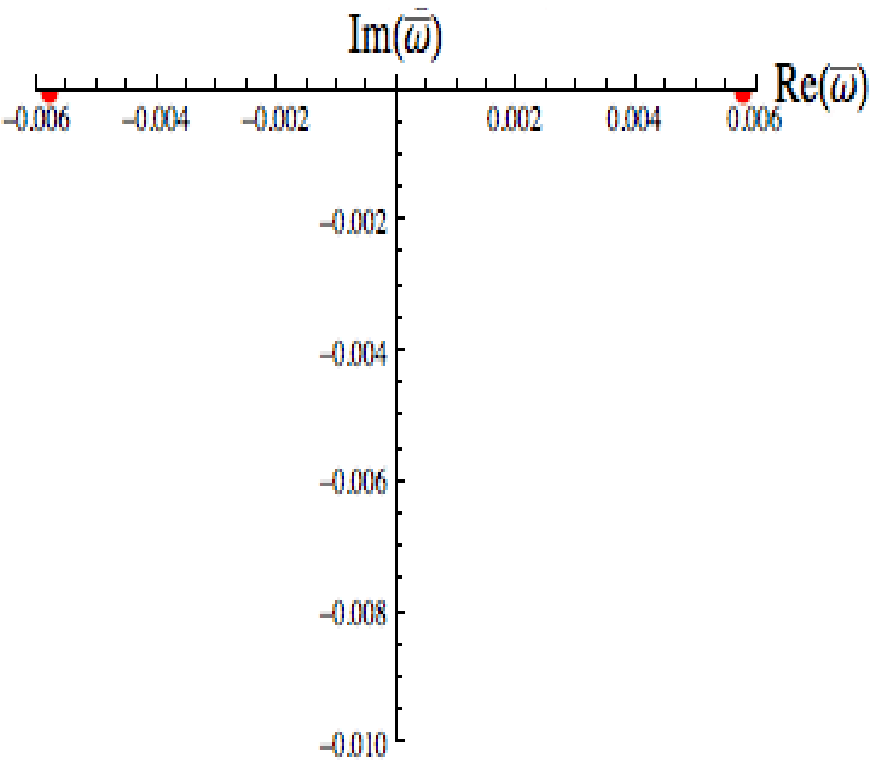
They form two purely imaginary modes. One of these is long-lived – it is the diffusion mode.

But there is no hydrodynamic sound mode.

This is because hydrodynamic sound is an excitation in the energy density Green's function – and we have explicitly suppressed these fluctuations due to our $N_f \ll N_c$ limit.

The D3/D7 Theory Sound Mode

Is there a crossover from collisionless to hydrodynamic behaviour at even higher temperatures?



The zero sound modes become less stable and collide at a sufficiently high temperature.

They form two purely imaginary modes. One of these is long-lived – it is the diffusion mode.

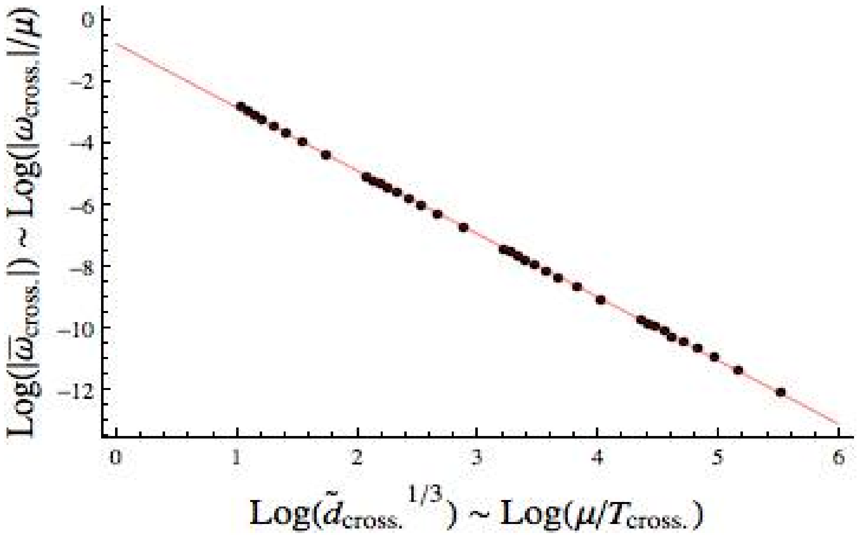
But there is no hydrodynamic sound mode.

This is because hydrodynamic sound is an excitation in the energy density Green's function – and we have explicitly suppressed these fluctuations due to our $N_f \ll N_c$ limit.

The D3/D7 Theory Crossover

At what temperature does this collisionless/hydrodynamic crossover occur?

Define the crossover temperature to be when the sound poles collide on the purely imaginary axis. How does it vary with the frequency?



$$\log(\omega_{\mathrm{cross.}}/\mu) \sim -2\log(\mu/T_{\mathrm{cross.}})$$

$$\longrightarrow \quad \tau \equiv 1/\omega_{\mathrm{cross.}} \sim \mu/T^2$$

for both massless and massive hypermultiplets

This is exactly the same as for a Landau-Fermi liquid, where the interpretation is that the quasiparticles have a lifetime ~μ/T^2 before which they decay due to thermal collisions.

The D3/D7 Theory Summary

In summary, the collective bosonic excitations behave exactly as expected of a Landau-Fermi liquid, except for the technical limitation on hydrodynamic sound.

This is surprising because it is not a Landau-Fermi liquid:
• Its heat capacity is $C \propto T^6$
• Its entropy is non-zero when T=0.
• There appears to be no direct signature of a Fermi surface in the bosonic correlators.

What is going on? There are various possibilities
• We are not studying the true ground state of the theory.
• There is a Fermi surface, but it has yet to be discovered.
• This is a new type of (strongly-coupled) quantum liquid with no Fermi surface, but which has a zero sound mode identical to that of a Landau-Fermi liquid.

???

The RN-AdS$_4$ Theory

A simpler question is are these properties common to holographic field theories at high density?

Generically, the corresponding gravitational theories will be solutions to supergravity with a non-trivial gauge field.

The simplest consistent truncation of supergravity theories with a non-trivial gauge field is Einstein-Maxwell theory, which has a charged Reissner-Nordstrom black hole solution (RN-AdS$_4$)

$$S \sim \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right)$$

Fermionic correlators in this theory do not behave like those of a Landau-Fermi liquid.

The low energy properties of this theory are governed by the AdS$_2$xR^2 near-horizon region of the black hole.

Faulkner et. al.
arXiv:0907.2694 [hep-th]

The RN-AdS$_4$ Theory Sound Mode

Does this theory have a 'zero sound' mode? What are its properties?

There is a stable sound mode at T=0 when $\omega, q \ll \mu$

$$\omega = \pm \frac{q}{\sqrt{2}} - i0.083\frac{q^2}{\mu} + \dots$$

Edalati, Jottar & Leigh
arXiv:1005.4075 [hep-th]

How does the attenuation behave as the temperature is increased?

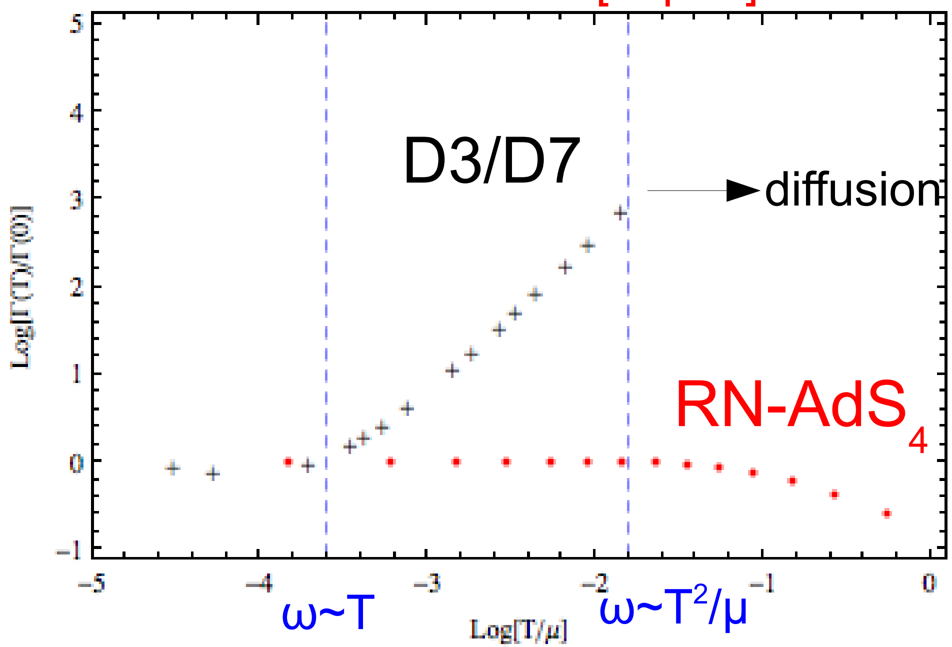
Is there a crossover between collisionless and hydrodynamic behaviour as the temperature is increased?

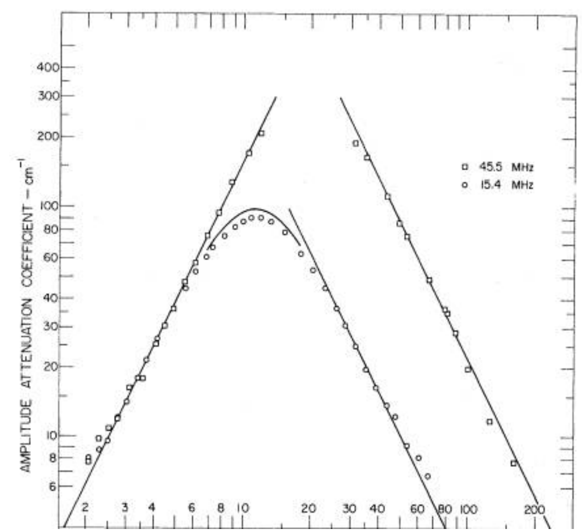
Note that we are no longer stuck with the probe limit – the charge density and energy density are coupled.

The RN-AdS$_4$ Theory Sound Mode

How does the attenuation behave as the temperature is increased?

R.D. and N. Kaplis
arXiv:1110.xxxx [hep-th]





Liquid Helium-3

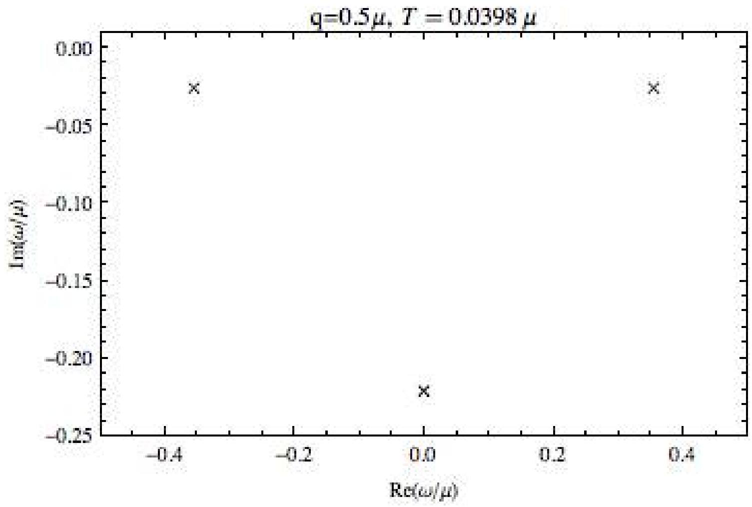
When T<<μ, the decay rate of the sound mode is essentially featureless.

It does not behave anything like the D3/D7 or Landau-Fermi liquid zero sound mode.

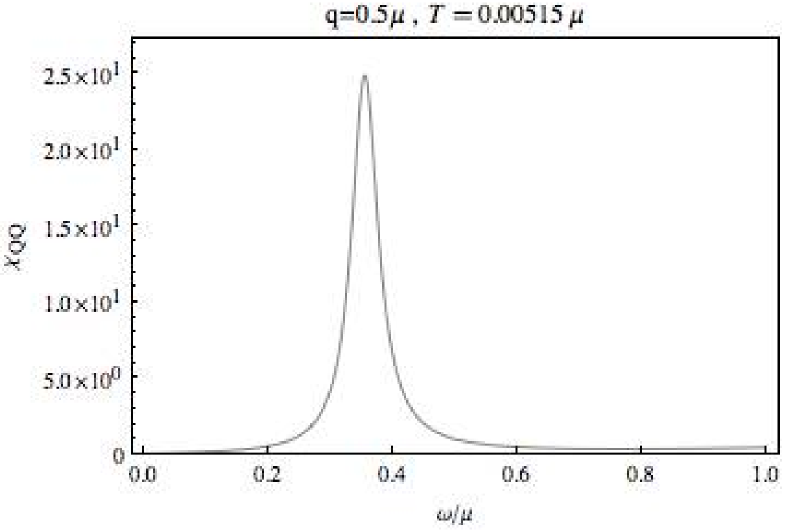
The RN-AdS$_4$ Theory Crossover

Is there a crossover between collisionless and hydrodynamic behaviour as the temperature is increased?

Consider the full range of temperatures from T<<ω,q<<μ to ω,q<<μ<<T



Poles of charge density correlator

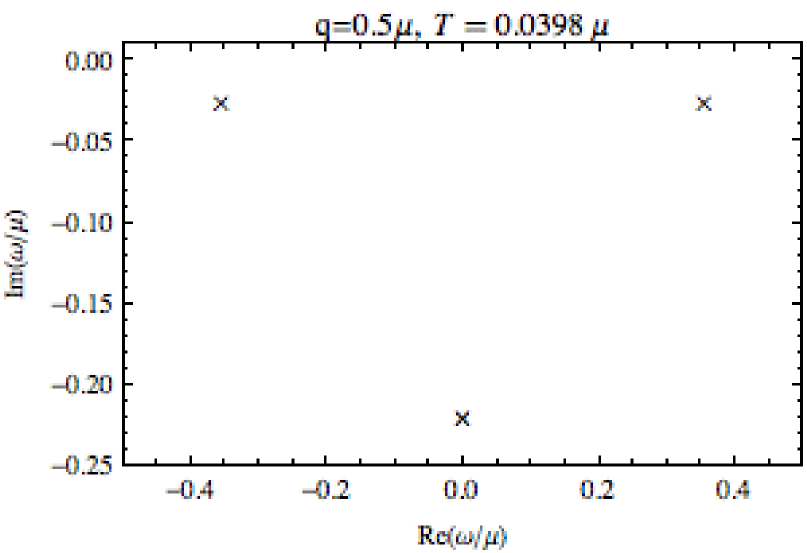


Imaginary part of charge density correlator

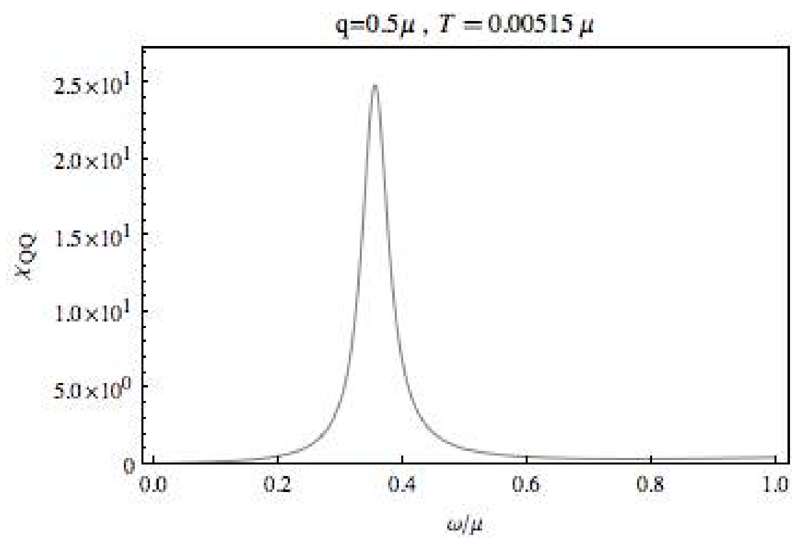
The RN-AdS$_4$ Theory Crossover

Is there a crossover between collisionless and hydrodynamic behaviour as the temperature is increased?

Consider the full range of temperatures from T<<ω,q<<μ to ω,q<<μ<<T



Poles of charge density correlator



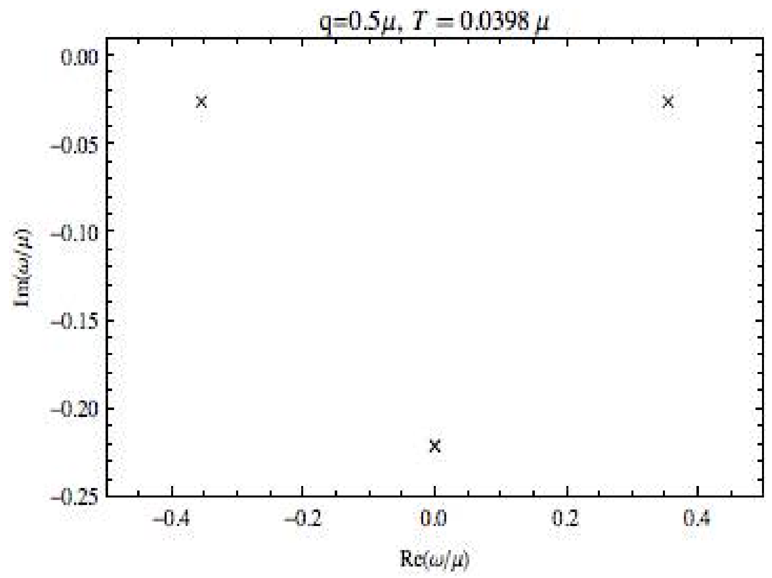
Imaginary part of charge density correlator

The sound and diffusion modes are present at all non-zero T.
The sound mode always has a longer lifetime.

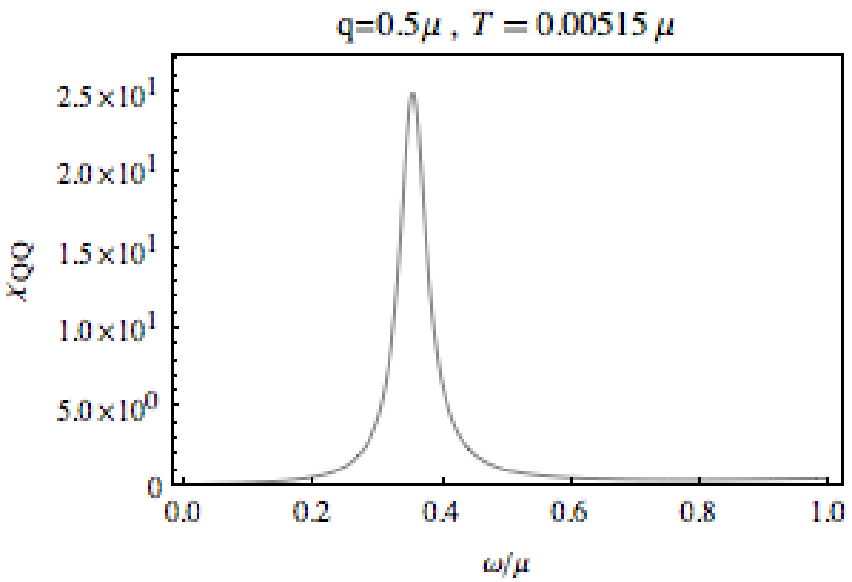
The RN-AdS$_4$ Theory Crossover

Is there a crossover between collisionless and hydrodynamic behaviour as the temperature is increased?

Consider the full range of temperatures from T<<ω,q<<μ to ω,q<<μ<<T



Poles of charge density correlator



Imaginary part of charge density correlator

There is a change in the spectral function when T~μ^2/q>>μ.
It occurs due to the changing residues of the poles.

The RN-AdS_4 Theory Summary

There are sound and diffusion-like modes at all non-zero temperatures.

The attenuation of the sound mode does not behave like that of a Landau-Fermi liquid, in line with the other properties of the theory.

There is a crossover at very high temperatures from the sound peak dominating the charge density spectral function to the diffusion peak dominating.

This occurs due to the changing residues at each pole. We know of no simple interpretation of this analogous to the Landau-Fermi liquid explanation for D3/D7.

It exhibits another way in which such crossovers can occur.

A Few Open Questions

There are lots more holographic theories that support sound at T=0 and large density.
What are their thermal properties?
Is there some universality between different theories?

We could not access the D3/D7 hydrodynamic sound mode due to the probe limit.
Can we get around the limit in this theory, or similar theories?
Does the coupling to the energy-momentum tensor reproduce the full Landau-Fermi liquid sound attenuation curve?
Or does it change the probe results in the collisionless regimes?

The RN-AdS$_4$ theory was chosen for simplicity.
How much do its properties depend upon the exact choice of supergravity truncation?