EMPG SEMINAR
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①.

Linear Sigma Models for Heterotic Vacua
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with D. Morrison, I. Melnikov, R Plesser, M Kruezer., S Seth. — 1103.1322., 1004.5447, 1001.2104, 0010:0012., 0712.3272. — Review: 1010.4667.

use ST to study apps of het. strg.

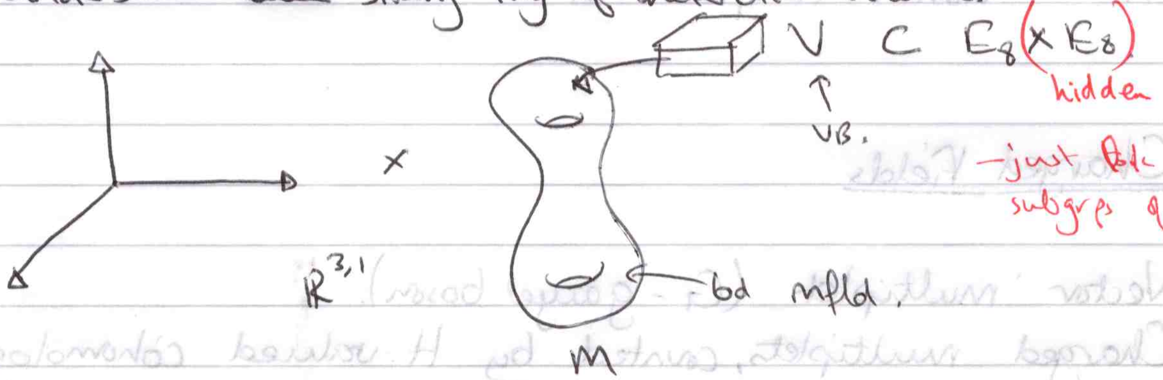
Heterotic vacua important.
→ Phenomenology (easy realise gauge thys GUT SU(5))
→ Mathematics (type II → mirror symmetry about CY$_3$ ~~&~~ new facts about CY$_3$ & vector bundles).

SUGRA : $\frac{R}{l_s} \to \infty$

→ well understood.

String thy → little known.

AIM : Understand ~~abou~~ string thy of heterotic vacua.



– just look @ subgrps of E_8.

(N=1 susy).

EOMs → ① $R_{i\bar{j}} = 0 \longrightarrow M = CY_3$

② $F_{(0,2)} = F_{(2,0)} = 0$. (V is holomorphic).

③ $g^{i\bar{j}} F_{i\bar{j}} = 0$ HYM $\xleftrightarrow{\text{DUY}}$ V "stable".

hermitian Yang Mills

Vacuum is anomaly free for $ch_2(M) = ch_2(V)$. $\left(\Leftrightarrow dH_3 = \mathrm{tr}\, R \wedge R - \mathrm{tr}\, F \wedge F \right)$

What is low energy theory? $N=1$ chiral gauge thy.

If H is the structure group of V then the gauge group

$G = [H, E_8]$

eg $H = SU(3)$, $G = E_6$

$H = SU(4)$, $G = SO(10)$

$H = SU(5)$, $G = SU(5)$.

FIELD CONTENT

→ Neutral fields

① Gravity multiplet
② $h^{1,1} + h^{2,1}$ neutral chiral multiplets. (Kahler + complex st.).
③ $h^{bun} = \dim H^1(M, \mathrm{End}\, V)$.

→ Charged Fields

④ Vector multiplet (G-gauge boson).
⑤ Charged multiplets, counted by H-valued cohomology groups.

~~$H = SU(3)$~~

Eg $H = SU(3)$, $G = E_6$

$h^{1,1} = \dim H^1(M, V^*)$
multiplets in $\overline{27}$ of E_6.

$h^{2,1} = \dim H^1(M, V)$ in 27 of E_6.

$\mathcal{L} = \mathcal{L}_{KE} + \mathcal{L}_{4uk} + \ldots$

$\mathcal{L}_{4uk} = $ (topological invariants)

$(\overline{27})^3 = \int_M J \wedge J \wedge J + \ldots$

quantum corrected.
How to compute this?

Examples & Expectations

→ $g_s \to 0$, α' dominates

→ "Standard ~~E~~ embedding"

Choose $V = TM$ $[A = \omega]$.

tangent bundle.

• well understood.

Wonderful Results About Standard Embedding

① Compute Yukawas Exactly.

$$(\overline{27})^3 = 5 + 28759 + \dots + c_n q^n t$$

$c_n \in \mathbb{N}$. $\qquad$ (H)? $\quad q = e^{-V.l + ib}$

ws. instantons.

c_n count "rational curves" in M = Gromov-Witten.

Question $V \neq TM$.

Learn about V, M?
Generalisation of GW to V? } Yes: similar expansion.

② ~~(2,2)~~ $V = TM$ is described by (2,2) SCFT
Come in pairs $\longrightarrow$ Mirror Symmetry.

Given CY_3 M, $\exists$ M° with $h^{1,1} \leftrightarrow h^{2,1}$.

mirror partner

Batyrev-Borisov made "precise"

At the time: ~ 5 examples of CY_3.

Question: $V \neq TM$.
If V is def^n of TM &
M is "Reflexively Plain"
$\exists$ mirror map $(M, V) \leftrightarrow (M^\circ, V^\circ)$.

3 Moduli Space Metrics & Singularities.

$$\mathcal{M} = \mathcal{M}_{cs} \times \mathcal{M}_{\text{KAHLER}}.$$

Metric computed exactly - determined in terms of prepotential $\mathcal{F}$

Related to Yukawa.

QUESTION: Now have $\mathcal{M}_{bun}$.

Punchline if V in deformation of ~~TM~~ TM.

$$\mathcal{M} = \underbrace{\mathcal{M}_{(cs,T)} \times \mathcal{M}_{(\text{kahler},E)}}_{\text{bundle parameters.}}$$

* ∃ loci where they in singular
V = TM → conifold transition (D-branes get light).
V ≠ TM → now have V, what are singularities, parametrise them, topology change?

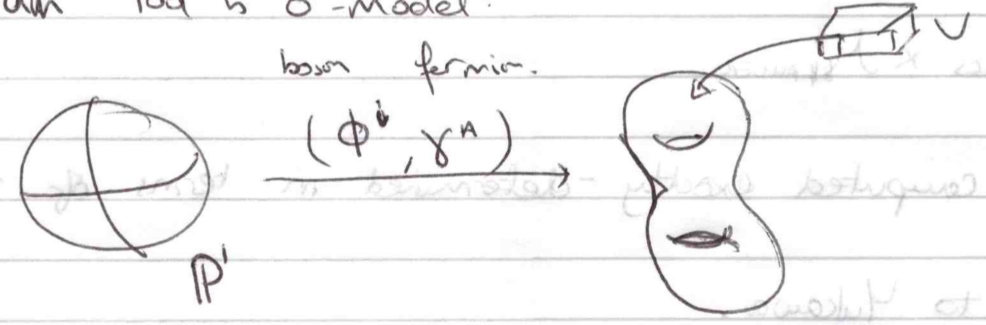
(This is a)
- Controlled setting for studying these models.

$(2,2) \longrightarrow (0,2)$.

→ even though susy is reduced to (0,2), still have computational control.

How?

Main tool is σ-model:



ϕ^i = coords of M.
γ^A = sections ~~coords~~ of V.

- NLSM $\longrightarrow$ hard (non-linear).

GLSM $\longrightarrow$ RG flow (linearises interactions).

× GLSM

NLSM $\rightsquigarrow$ × (flow)

× SCFT.

Builds CICYs with monad bundles.

★ 2d abelian gauge thy $U(1)^r$ w/ (0,2).

$$\left.\begin{array}{l} \Phi^i = \phi^i + \theta^+ \underset{\uparrow TM}{\psi^i} + \ldots \\ \Gamma^A = \underset{\nwarrow V}{\gamma^A} + \ldots \end{array}\right\} \quad \begin{array}{ll} Q^a_i & a = 1, \ldots, r. \\ & i = 1, \ldots, n. \\ Q^a_A & A = 1, \ldots, N. \end{array}$$

$\Sigma^a = \mathcal{O}^a + \cdots$ (twisted chiral multiplet)

rep^n of $H^1(M, V^*)$.

$\mathcal{L} = \mathcal{L}_{KE} + \mathcal{L}_J$.

Vacua: $\{P(\phi) = 0\} \subset \{ \underbrace{\sum_i Q_i^a |\phi^i|^2 - r^a}_{U(1)^r} = 0 \}$

hypersurface.

toric variety.

fermion γ^A

$$0 \to \mathcal{O}^r \xrightarrow[\text{"linear map"}]{E^A} \oplus \mathcal{O}(d_i) \xrightarrow{J_A} \mathcal{O}(D) \to 0.$$

polynomial

· (E, J) ~~are~~ holomorphic parameters of V.

① $N(M,V) = N^{2,2}(M,V) + N^{(0,2)}(M,V)$

$h^{1,1} + h^{2,1}$

$\sim \dim H^1(M, \mathrm{End}(V))$

A spin-wall for geometric calculation.

Then map

$$(P, J, q, E) \longleftrightarrow (P^\circ\ J^\circ, q, E^\circ).$$

$$(M, V) \longleftrightarrow (M^\circ, V^\circ)$$

② Yukawa Coupling

$$\overline{27}^3 = \sum C_n(E_\alpha) q^n$$

rational fns of parameters E,

$$C_n = \sum C_{n\alpha} E^\alpha$$

integers.

What are C_n?

→ $(\overline{27})^3$ doesn't depend on J,P

→ $(27)^3 = g(J,P)$. just J,P

$M_{(q,E)} \times M_{J,P}$.

What about beyond deformations of TM?

* For deformations of $TM \oplus \mathcal{O}$, $TM \oplus \mathcal{O} \oplus \mathcal{O}$, get SU(4) & SU(5) bundles V

Show GLSM makes sense (compute spectrum).

Should be able to compute Yukawas & moduli space properties.