Thermal Stability of Quantum Horizons

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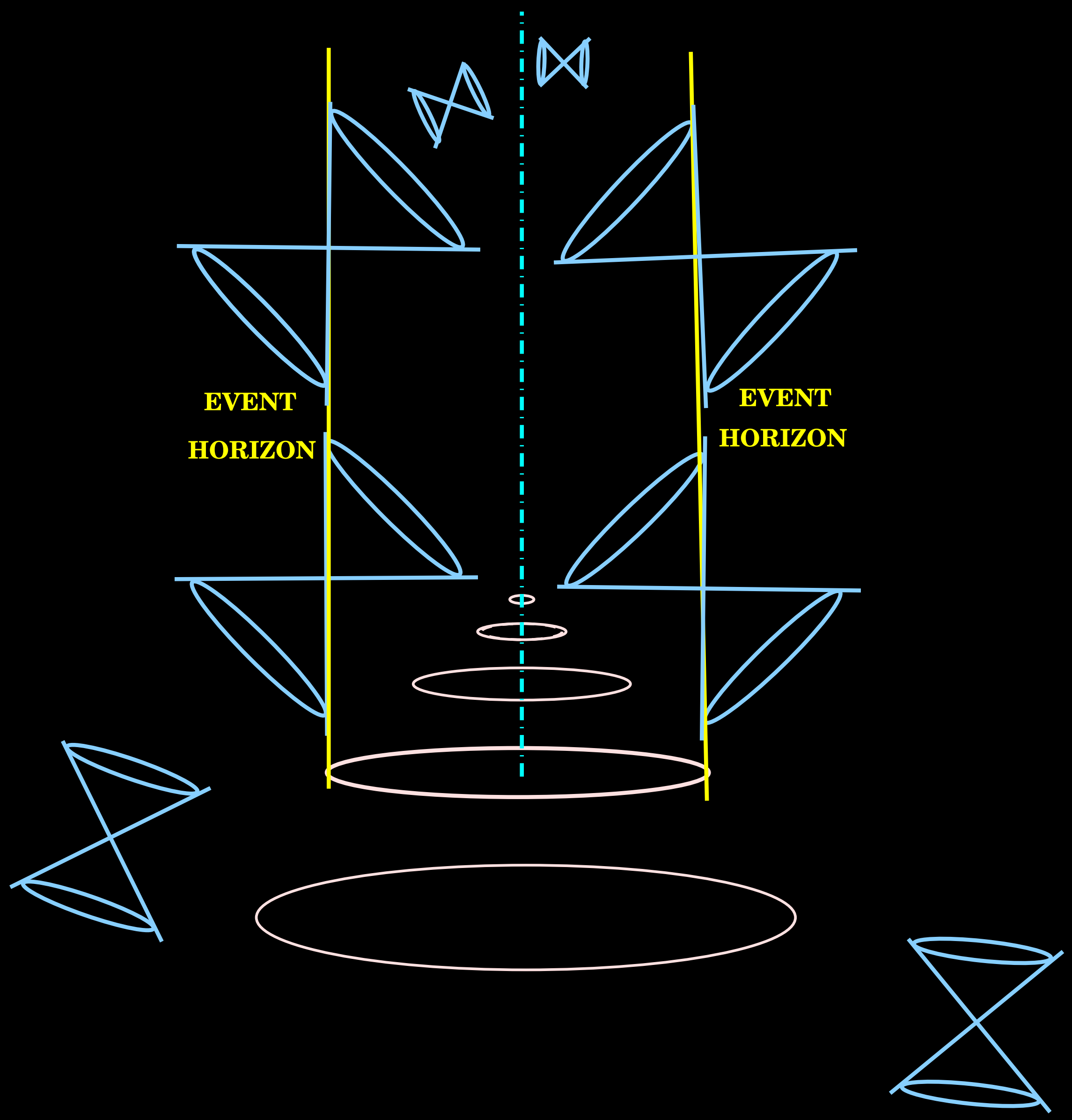
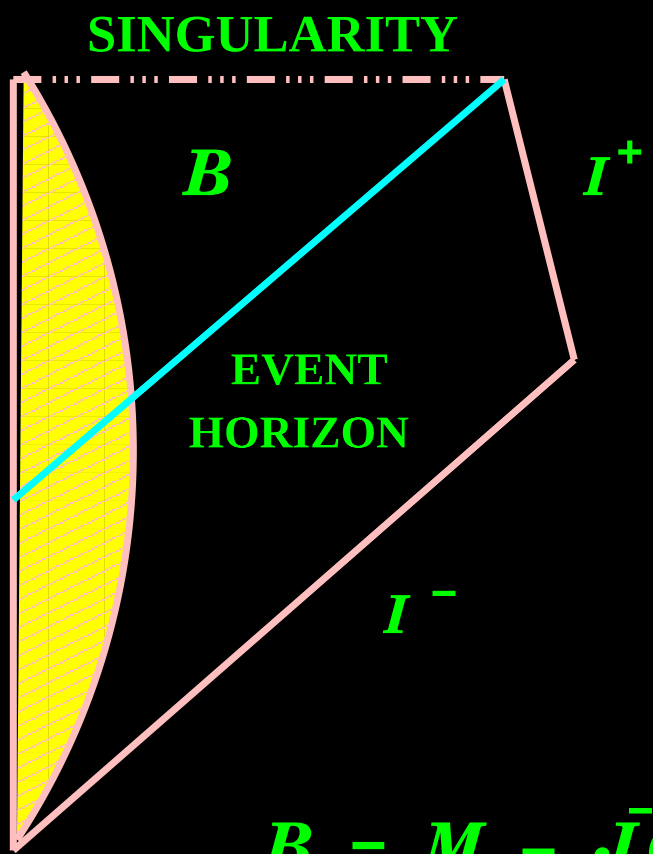
Spherical star collapsing to black hole Eddington-Finkelstein



Collapsing spherical star Penrose-Carter



$$B = M - \bar{J}(I^{+})$$

Black holes ... are the most perfect macroscopic objects there are in the universe. The only elements in their construction are our notions of space and time ... and because they appear as ... family of exact solutions of Einstein's equation, they are the simplest objects as well. - Subramanian Chandrasekhar

Yet Black hole sptms have

- **Event horizon : boundary of domain of communication**
- **Singularities, where all known laws of physics break down**

Laws of bh mech Bardeen, Carter, Hawking 1972

$$\begin{aligned} \delta A_{hor} &\geq 0 \\ \kappa_{hor} &= const \\ \delta M &= \kappa_{hor}\, \delta A_{hor} + \Phi \delta Q_{hor} + \cdots \end{aligned}$$

Gen. Sec. Law of thermo. **Bekenstein, 1973 :** $\delta(S_{out} + S_{bh}) \geq 0.$

Black holes radiate like a black body with a temperature $T_H = \hbar\kappa_{hor}$

Hawking 1974

Bekenstein-Hawking entropy

$$S_{bh} = \frac{\mathcal{A}_{hor}}{4l_P^2}\,(k_B = 1)$$

$l_P \equiv (G\hbar/c^3)^{1/2} \sim 10^{-33}cm \rightarrow$ quantum gravity

Need to go beyond classical GR - compulsion, not aesthetics

$S_{bh} \sim l_P^{-2} \rightarrow$ **nonperturbative QG**

Physics at 10^{-33} cm determines entropy of bh of size 10^{11} cm – Extreme Macro QM!

Issues to be addressed:

- **How is it that** $S_{bh} = S_{bh}(\mathcal{A}_{hor})$ **while** $S_{thermo} = S_{thermo}(vol)$ **?**
- **Why do some black holes thermally radiate or accrete incessantly ? (instability)**
- **What degrees of freedom contribute to** S_{bh} **?**
- **How do they lead to the B-H Entropy ?**

Outline

- **Thermal holography**
- **Quantum Isolated Horizon as equilibrium configuration**
- **Grand canonical and microcanonical entropy :** interplay of quantum spacetime and thermal fluctuations
- **Thermal Stability Criterion**
- **Isolated Horizon dof and dynamics**
- **LQG basics**
- **Quantum IH entropy**
- **Speculation : quantum origin of Chandrasekhar bound**
- **Pending Issues**

Electrodynamics in Minkowski sptm: Define charge holographically

$$Q(V) \equiv \int_{S=\partial V} \vec{E} \cdot \hat{n}\, d^2a$$

But, $\mathcal{H}_v = (1/8\pi)(\vec{E}^2 + \vec{B}^2) \rightarrow$ photons

Vac GR : no $\mathcal{T}^{ab}$ s.t. $\nabla_a \mathcal{T}^{ab} = 0$ in bulk

$$H_v \;=\; \int_{\mathcal{S}} [N\mathcal{H} + \mathbf{N} \cdot \mathbf{P}]$$

$$\approx\; 0 \text{ when } \mathcal{H} \approx 0,\; \mathbf{P} \approx 0$$

$\Rightarrow$ **no analogue of** $\mathrm{E}^2 + \mathrm{B}^2$ **in vac GR! Excitations 'polymeric'**

Grav energy globally defined

$$H_{Komar} = \frac{1}{8\pi}\int_{\mathcal{S}_\infty} d^2\sigma^{ab}\nabla_a K_b$$

Classically, bulk $\Rightarrow$ boundary entirely

Holography: 3 dim bulk info encoded on 2 dim bdy

Gravitons ?

Weak field approx $g_{ab} = \underbrace{\bar{g}_{ab}}_{bkgd} + \underbrace{h_{ab}}_{graviton}$

$$\mathcal{H}_v = (1/8\pi)[({}^3h)^2 + ({}^3\pi)^2]$$

As $|h| \nearrow$, $bkreactn$ $\nearrow$ approxn. invalid nonperturbatively

Quantum General Relativity

In general there are indep qu fluct on bdy : $\mathcal{H} = \mathcal{H}_v \otimes \mathcal{H}_b$

$$|\Psi\rangle \;=\; \sum_{v,b} c_{vb} \underbrace{|\psi_v\rangle}_{blk} \underbrace{|\chi_b\rangle}_{bdy} \in \mathcal{H}_v \otimes \mathcal{H}_b$$

$$\hat{H} \;=\; \underbrace{\hat{H}_v}_{blk} \otimes \mathbf{1} \;+\; \mathbf{1} \otimes \underbrace{\hat{H}_b}_{bdy}$$

Hamiltonian constraint (bulk)

$$\hat{H}_v\,|\psi_v\rangle \;=\; [\hat{H}_{EH,v} + \hat{H}_{MAX,v}]|\psi_v\rangle = 0$$

$$\hat{Q} = \hat{Q}_v \otimes \mathbf{1} + \mathbf{1} \otimes \hat{Q}_b$$

$$\hat{Q}_v|\psi_v\rangle = 0$$

New Hamiltonian constraint

$$\hat{H}'_v|\psi_v\rangle = 0$$
$$\hat{H}'_v \equiv \hat{H}_v - \Phi\hat{Q}_v$$

Grand Partition Function Majhi, PM 2011

$$Z_G = Tr\ \exp -\beta\hat{H}_T + \beta\Phi\hat{Q}$$
$$= \sum_{v,b} |c_{vb}|^2 \langle\chi_b| \otimes \langle\psi_v| \exp -\beta\hat{H}'|\psi_v\rangle \otimes |\chi_b\rangle$$
$$\hat{H}' = \hat{H}_T - \Phi\hat{Q}$$

Observe

$$\hat{H}' = (\hat{H}'_v \otimes \mathbf{1} + \mathbf{1} \otimes \hat{H}'_b)$$
$$\hat{H}'_v|\psi_v\rangle = 0$$

$$Z_G = Z_{Gb}$$
$$Z_{Gb} = Tr_b \exp -\beta(\hat{H}_b - \Phi \hat{Q}_b)$$

Bulk states decouple! Boundary states determine bh thermodynamics completely → **Thermal holography !** (PM 2001, 2007; Majhi, PM 2011)

Different from strong holography ('t Hooft 1992; Susskind 1993; Bousso 2002)

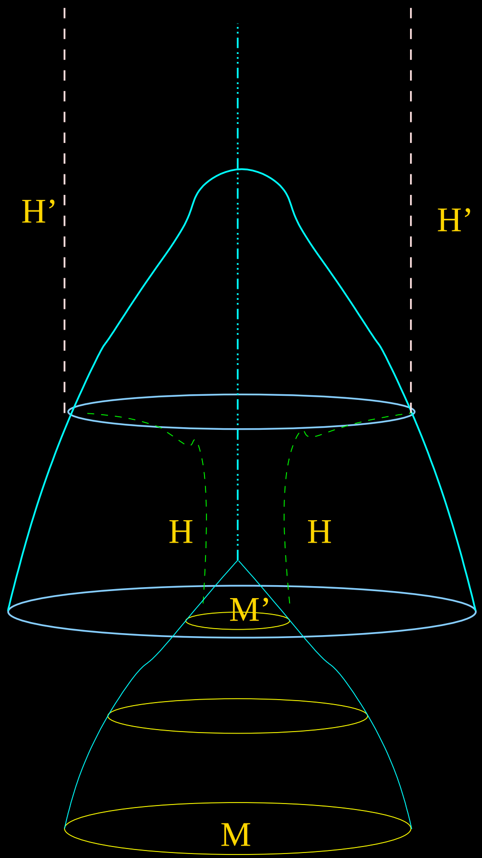
Holographic Hypothesis (HH)

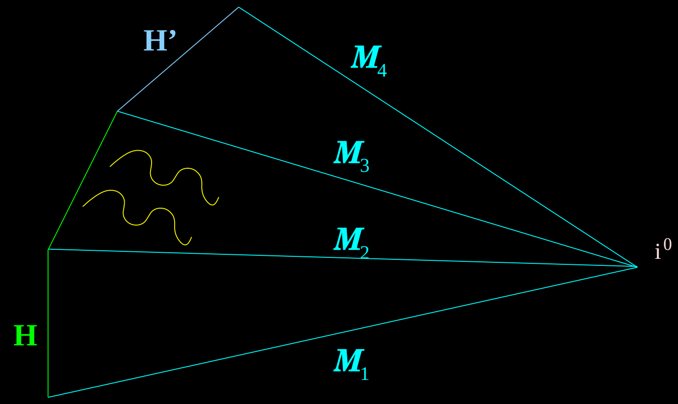
... Given any closed surface, we can represent all that happens (gravitationally) inside it by degrees of freedom on this surface itself. This ... suggests that quantum gravity should be described by a **topological** *quantum field theory in which all (gravitational) degrees of freedom are projected onto the boundary.*

In contrast, ours underlines primacy of boundary states for bh thermodyn

What sort of boundary ? Not asymptotic bdy; not Event Horizon → **teleo-logical, globally stationary, ...**)

Work with Isolated Horizons (IH) as local, non-stationary, equilibrium generalization of EHs (Ashtekar et. al. 1997-2001)





- **Nonstationary**
- Null (lightlike) inner boundary of sptm with topol $R \otimes S^2$
- Marginally Outer Trapped : $\theta_{(l)} = 0\ ,\ \theta_{(n)} < 0$
- $\mathcal{A}(S^2) = const \rightarrow$ isolation
- *Zeroth law of IHM* surface grav $\kappa_{IH} = const$
- Possible to define mass on IH : $M_{IH} = M_{IH}(A, Q)$
- $M_{IH} \equiv M_{ADM} - \mathcal{E}^{\infty}_{rad}$ s.t. $\delta M_{IH} = \kappa \delta \mathcal{A}_{hor} + \Phi \delta Q_{hor}$ *(Ist law of IHM)*
- IH is *microcanonical ensemble* with fixed $\mathcal{A}_{hor}, Q_{hor}$
- Hawking radiation requires IH $\rightarrow$ Dynamical Hor

Grand Canonical Ensemble of IHs in rad bath : compute $Z_b \rightarrow S_{can}$

- Assume equil. IH with fixed $\mathcal{A}_{IH}, Q_{IH}$ and $M_{IH} = M(\mathcal{A}_{IH}, Q_{IH})$.
- Keep Gaussian fluct. (Das, Bhaduri, PM 2001; Chatterjee, PM 2003)
- $\mathcal{A}_n \sim n l_P^2, \; n >> 1$ (justify later)

$$S_{can}(\mathcal{A}_{IH}) \;=\; S_{IH}(\mathcal{A}_{IH}) \;+\; \underbrace{\frac{1}{2}\log\Delta(\mathcal{A}_{IH})}_{th\ fluc\ corr}$$

Two issues arise :

- Expect S_{can} $+ve\ real \;\Rightarrow\; C\; >\; 0$ (th stab). How/when violated (e.g. Schwarzschild, RN)?
- How to compute S_{IH} ? Need quantum theory of IH

Condition for thermal stability (Majhi, PM 2011)

$$Z_G = \sum_{m,n} g(A_m, Q_n) \exp -\beta[M(A_m, Q_n) - \Phi Q_n]$$

LQG : $A_m \sim m l_P^2 \ , \ m >> 1 \ , \ A_{hor} >> l_P^2 \ , \ Q_n \sim n$

$$\begin{aligned} Z_G &= \int \frac{dA\, dQ}{A_x\, Q_y} \, g(A, Q) \ \exp -\beta[M(A, Q) - \Phi Q] \\ &= \int dA\, dQ \; e^{S(A) - \beta M(A,Q) + \beta \Phi Q} \end{aligned}$$

$S(A) \equiv \log g$ assumed indep of Q. Measure factors do not contribute.

Expansion around $\bar{A}, \bar{Q}$ $\Rightarrow$: Gaussian approx

$\Rightarrow$ conditions on Hessian matrix for convergence of Z_G

$$\beta \equiv \frac{S_A(\bar{A})}{M_A(\bar{A},\bar{Q})} > 0$$

$$\beta M_{AA}(\bar{A},\bar{Q}) - S_{AA}(\bar{A}) > 0$$

$$\left\{\beta M_{AA}(\bar{A},\bar{Q}) - S_{AA}(\bar{A})\right\} \beta M_{QQ}(\bar{A},\bar{Q}) - \beta^2 M_{AQ}^2(\bar{A},\bar{Q}) > 0$$

Solve as Partial differential inequality

Ansatz:

$$M(\bar{A},\bar{Q}) = \mu(\bar{A}) \cdot \chi(\bar{Q}) \ , \ \chi(0) = 1$$

$\rightarrow$

$$\frac{\mu}{\mu_A}\left[\frac{\mu_{AA}}{\mu_A} - \frac{S_{AA}}{S_A}\right] > \frac{\chi_Q}{\chi}\frac{\chi_Q}{\chi_{QQ}}$$

Solution :

$$\chi(Q) = (1 + CQ)^{\frac{1}{\kappa-1}}$$
$$\mu(A) > (\alpha S)^{\frac{\kappa}{\kappa-1}}$$

Choose constants $\kappa > 1$, $(k_B \alpha)^{\frac{\kappa}{\kappa-1}} = M_P \Rightarrow$

$$\frac{M}{M_P} > \frac{S}{k_B}\left[\frac{S}{k_B(1+CQ)}\right]^{\frac{1}{\kappa-1}}$$
$$= \frac{S}{k_B}\left[1 + \frac{1}{\kappa - 1}\ln\left(\frac{S}{k_B(1+CQ)}\right) + \cdots\right] > \frac{S}{k_B}$$

- Checks out with $Q = 0$ case PM 2007, 2009
- **No classical metric used in derivation**
- **Necessary and sufficient condition for qbh to be thermally stable**

Fiducial checks Majhi, PM 2011

Reissner Nordstrom

$$M_{RN} = \left(\frac{A}{4\pi}\right)^{1/2} \left[1 + \frac{4\pi Q^2}{A}\right]$$

Violates stability bound $\rightarrow$ thermally unstable

Anti-de Sitter Reissner Nordstrom

$$M_{ADSRN} = \left(\frac{A}{4\pi}\right)^{1/2} \left[1 + \frac{4\pi Q^2}{A} + \frac{A}{4\pi l^2}\right]$$

Satisfies stability bound for $A >> 4\pi l^2$

- **Can distinguish 'energy driven' vs 'entropy driven' processes**
- **Criterion holds for all isolated horizons**
- **Corrections to area law crucial for nontriviality of bound**

Canonical GR as gauge theory of real $SU(2)$ connections Sen 1982; Ashtekar 1985, 1996; Barbero 1995; Immirzi 1997

Spatial slices $\mathcal{S}_t : t(x) = const$ with congruence $t^a = N^a + Nn^a$, $n^a \to$ unit normal to $\mathcal{S}_t$

Phase space variables

$$(q_{ab}, \pi_{ab}) \to^{\text{canon transf}} (E^i_a \equiv \det e\, e^i_a \,,\ K^i_a \equiv q^b_a\, \omega^{0i}_b)$$
$$\to^{\text{canon transf}} (A^{(\gamma)i}_a \equiv \epsilon^{ijk}\omega_{a\ jk} + \gamma\, K^i_a \,,\ E^i_a) \,, \gamma > 0$$

where *time gauge* $e^0_a = -n_a = -N\partial_a t$ has been chosen $\Rightarrow$ local boosts frozen $\Rightarrow$ **residual** $SU(2)$ **gauge inv**

Canonical PB

$$\{A^{i(\gamma)}_a(x) \,,\ E^{bj}(y)\}_{PB} = \gamma\, G\, \delta^b_a\, \delta^{(3)}(x, y)$$

Canonical 1st Class constraints :

$$D_a(A^{(\gamma)})E^{ai} = \gamma\,\mathcal{G}^i \;\approx\; 0$$

$$E^{ai}F^i_{ab} = \gamma(\Pi_b + \gamma K^i_b \mathcal{G}_i) \;\approx\; 0$$

$$H(A,E) = (\det E)^{-1/2}[\epsilon_{ijk}E^{ai}E^{bj}F^k_{ab} \;+\; 2(1+\gamma^2)E^{[a|i}E^{|b]j}K^i_a K^j_b] \approx 0$$

On IH **null** bdy $\Rightarrow {}^3g_{ab}dx^a dx^b = 0 = {}^3g$

3 dim gravity : $\mathcal{S}_{IH} = \int_{IH} \sqrt{-{}^3g}\,{}^3R$ impossible!

On IH only possibility : 3 dim Topological gauge theory !

Consider e.g. Schwarzschild bh Kaul, PM 2011

$$ds^2 = -f(u,v)\ du dv\ -\ r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$f(u,v)\ =\ 4\frac{r_0^3}{r}\ \exp -\frac{r}{r_0}$$

$$uv\ =\ -\left(\frac{r}{r_0} - 1\right)\ \exp\frac{r}{r_0}$$

Tetrads :

$$e^0\ =\ f^{1/2}(u,v)\left(\frac{u}{\alpha}\,dv + \frac{\alpha}{v}\,du\right)$$

$$e^1\ =\ f^{1/2}(u,v)\left(\frac{u}{\alpha}\,dv - \frac{\alpha}{v}\,du\right)$$

$$e^2\ =\ r d\theta\ \ ,\ \ e^3 = r\sin\theta d\phi$$

Spin connections computed from

$$D_a\, e_b^I \equiv \nabla_a\, e_b^I - \omega_{aK}^I\, e_b^K = 0$$

Define $SU(2)$ connection (in time gauge)

$$A_a^i \equiv \frac{1}{2}\epsilon^{ijk}\,\omega_{ajk} - \gamma\,\omega_a^{0i}$$

Compute explicitly Curvature $F^i \equiv dA^i + \epsilon^{ijk}\,A_j \wedge A_k$ and solder 2-form $\Sigma^i \equiv \epsilon^{ijk}\,e_j \wedge e_k$

Pull back both forms to Event Horizon (null inner boundary) $\Rightarrow$

$$F^i \hat{=} -\frac{2\gamma\,G}{r_0^2}\,\Sigma^i$$

Chern Simons EoM on IH

$$\frac{k}{2\pi}\,F^i \hat{=} -\Sigma^i \;\;, \;\; k \equiv \frac{A_{IH}}{8\pi\gamma G}$$

Event (Isolated) horizon described by $SU(2)$ Chern Simons theory

$$\mathcal{S}_{IH}[\mathbf{A}] = tr \int_{IH} \epsilon^{abc} \left[\left(\frac{k}{2\pi} \right) (\mathbf{A}_a \partial_b \mathbf{A}_c + \mathbf{A}_a \mathbf{A}_b \mathbf{A}_c) + \mathbf{A}_a \mathbf{\Sigma}_{bc} \right]$$

$\mathcal{S}_{GR} + \mathcal{S}_{IH} \rightarrow$ variational principle OK, provided

$$\left(\frac{k}{2\pi} F_{CS} + \Sigma \right)_{S^2} = 0 \; , \; k \equiv (\mathcal{A}_{IH}/4\pi\gamma G) >> 1$$

$SU(2)$ **Chern Simons theory can be gauge fixed on IH to** $U(1)$ **Chern Simons theory** *provided* appropriate restrictions on sources Σ comp on IH are accounted for Basu,Kaul,PM 2010; Kaul,PM 2011

Loop Quantum Gravity/Canonical QGR (bkgd-indep, nonpert)

$SL(2,C)$ inv self-dual gravity $\rightarrow$ complex config. space $\rightarrow$ gauge fix to Barbero-Immirzi $SU(2)$ inv formlation

For $\mathbf{A}, E$ canonical quantization $\Rightarrow$

$$\left[\hat{A}^i_a(x)\ ,\ \hat{E}^b_j(y)\right] \ = \ i\hbar\, \delta^b_a\, \delta^i_j\, \delta^{(3)}(x,y)$$

To avoid singularity of CCR use Global variables : classically

$$\text{holonomies } h_l \ \equiv \ \mathcal{P} \exp \int_l \mathbf{A}$$

$$\text{Fluxes } \Phi_{f,S} \ \equiv \ 8\pi G\gamma \int_S d^2\sigma^{ab}\, \epsilon_{abc}\, f_i E^{ci}$$

Class config space $\mathbf{A} \equiv \{A\}/(gauge)$

Quant Config Space $\bar{A} \in \bar{\mathbf{A}} \rightarrow$ non-smooth (distributional) fields

Sample $\bar{\mathbf{A}}$ by finitely many probes : graph $\alpha = (n_e \text{ edges },\ n_v \text{ vertices})$ embedded in spatial slice S

Restrict $A \rightarrow A_\alpha$ to $\alpha \Rightarrow \bar{\mathbf{A}} \rightarrow \bar{\mathbf{A}}_\alpha$

n_e-tuples of holonomies $h_{e_1}[A_\alpha], ..., h_{e_{N_e}}[A_\alpha]$

Wave functionals

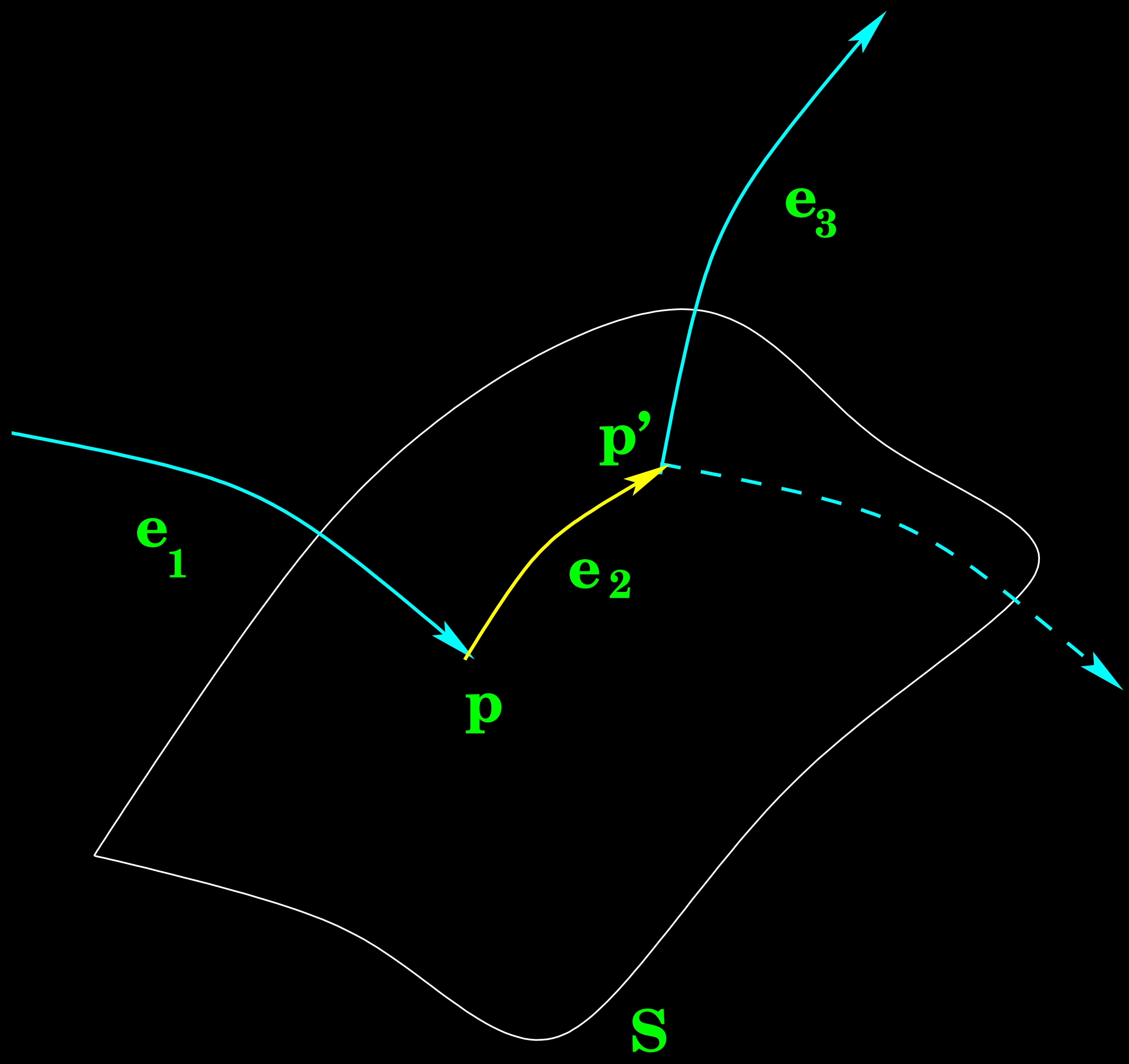
$$\Psi_\alpha[A_\alpha] \equiv \chi_\alpha(h_{e_1}, ..., h_{e_{n_e}})$$
$$\langle \Psi_\alpha \, , \, \tilde{\Psi}_\alpha \rangle = \int_{\bar{\mathbf{A}}} d\mu_\alpha \, \bar{\chi} \, \tilde{\chi}$$

Edges $e_I \, , \, I = 1, ..., n_e$ carry spin $j_I = 0, 1/2, 1, ...$; vertices carry $SU(2)$ invariant tensors, depending upon valence

LQG : promote holomies, fluxes to operators : $\hat{h}_l(\hat{\mathcal{A}}) \, , \, \hat{E}_{f,S} \rightarrow$

CCR among these, assuming edge e intersects S at vertex v,

$$\Big[h_e[A] \, , \, \Phi_{[S,f]}[E] \Big] = \frac{1}{2} k(e) \, h_e \, \tau \cdot f \text{ if } v \text{ source } e$$
$$= -\frac{1}{2} k(e) \, \tau \cdot f \, h_e \text{ if } v \text{ target } e$$



e3
p'
e1
e2
p
S

On Wave functionals, any function $O(A)$ acts by multiplication

$$O(A) \to \hat{O}\ \Psi_\alpha[A] = O(A)\Psi_\alpha[A]$$

Action of $\Phi_{S,f}[E]$

$$\begin{aligned}\hat{\Phi}_{S,f}[E]\ \Psi_\alpha &= \left[\Phi_{S,f}[E]\ ,\ \Psi_\alpha[A]\right] \\ &= \frac{\hbar}{2}\sum_{v\in S} f^i\ \left[J^{S,v}_{(u)i} - J^{S,v}_{(d)i}\right]\ \chi\alpha\end{aligned}$$

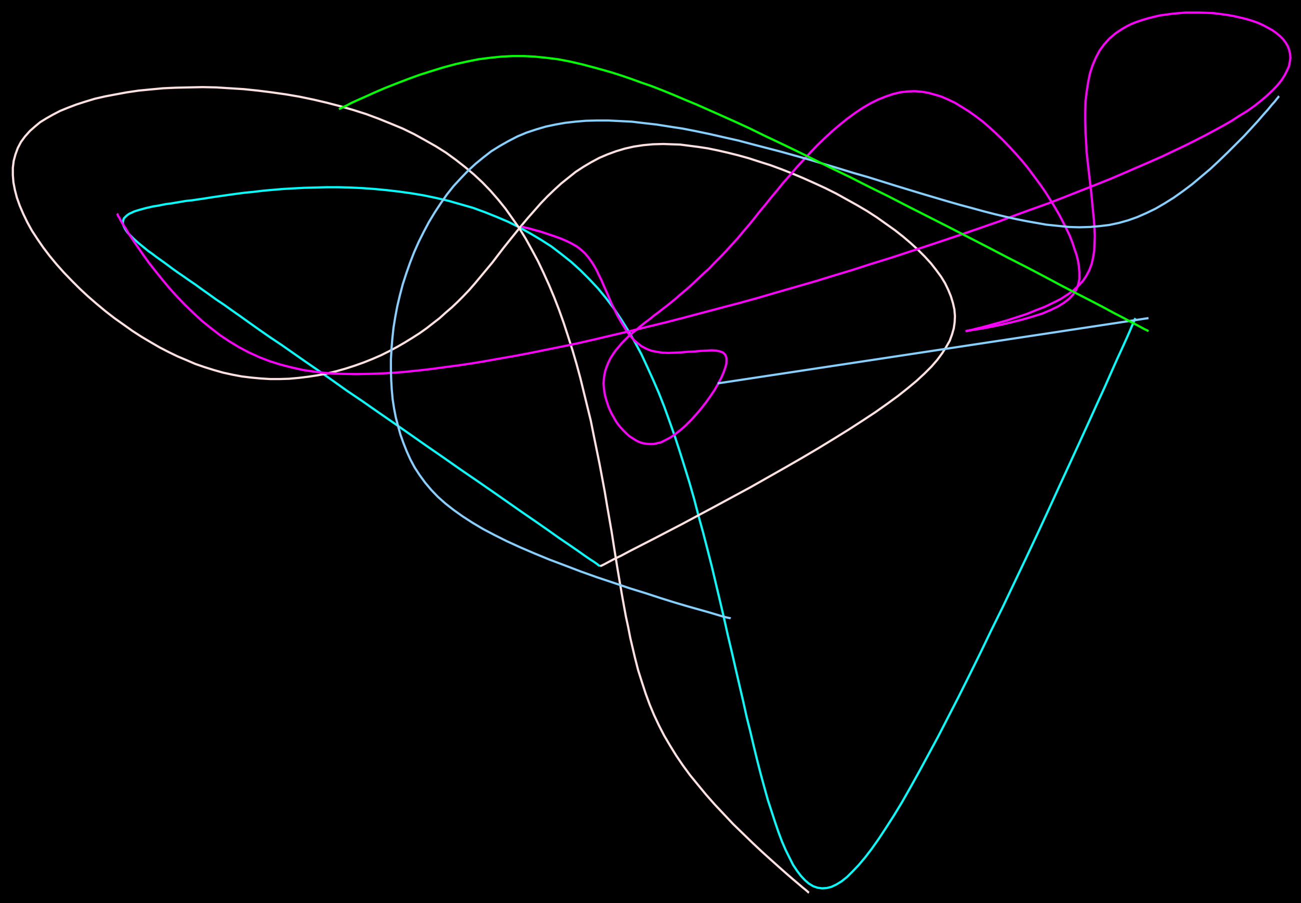
where,

$$\begin{aligned}J^{S,v}_{(u)i} &\equiv J^{e_1,v}_{(u)} + J^{e_2,v}_{(u)} + \cdots + J^{e_u,v}_{(u)} \text{ for edges above } S \\ J^{S,v}_{(d)i} &\equiv J^{e_{u+1},v}_{(d)i} + J^{e_{u+2},v}_{(d)i} + \cdots + J^{e_{u+d},v}_{(d)i} \text{ for edges below } S\end{aligned}$$

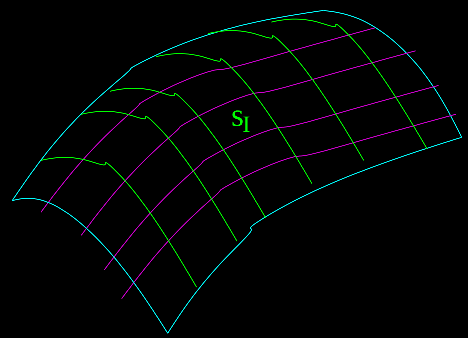
$\Rightarrow \Phi_S[f \cdot \Sigma]$ acts through spin operators

Spinnet states diagonalize spin operators $\Rightarrow$ observables expressible in terms of spin operators have exactly determined spectrum (area, volume ...)

Spin network : Quantum Space



Area operator (also volume, length) have bounded, discrete spectrum



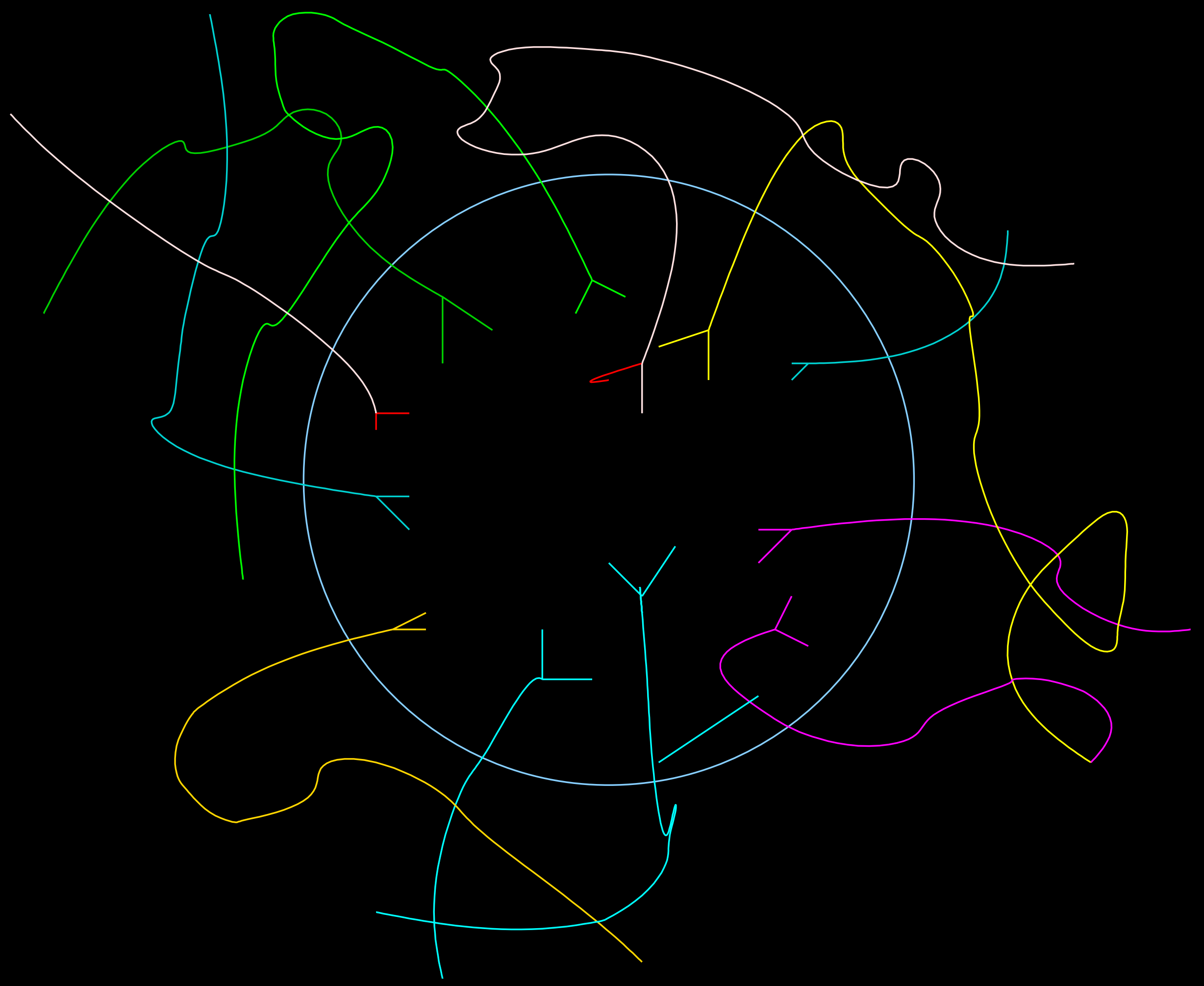
$$\hat{\mathcal{A}}_S \equiv \lim_{N\to\infty} \sum_{I=1}^{N} \int_{S_I} \det{}^{1/2}[{}^2g(\hat{E})]$$

$$\simeq \lim_{N\to\infty} \sum_{I=1}^{\infty} \sqrt{\Phi_{S_I}(\tau \cdot E)\Phi_{S_I}(\tau \cdot E)}$$

$$\hat{\mathcal{A}}_S\Psi = a(j_1, ..., j_N)\Psi$$

$$a(j_1, \ldots, j_N) = 8\pi\gamma l_P^2 \sum_{p=1}^{N} \sqrt{j_p(j_p+1)}$$

‘Quantum’ Isolated Horizon → effective description (Ashtekar, Baez, Corichi, Krasnov 1997)



Need to compute $S_{IH} = \log \dim \mathcal{H}_{CS+ptsources(j_1,...j_n)}$ for fixed $\mathcal{A}_{IH} \pm O(l_P^2)$

Witten (1986) : $\dim \mathcal{H}_{CS} = \dim \left[Inv \left(\otimes_p [j_p] \right) \right]$ where, $[j_p] \rightarrow$ conf current block of $SU(2)_k$ WZW on pth puncture of S^2_{IH}

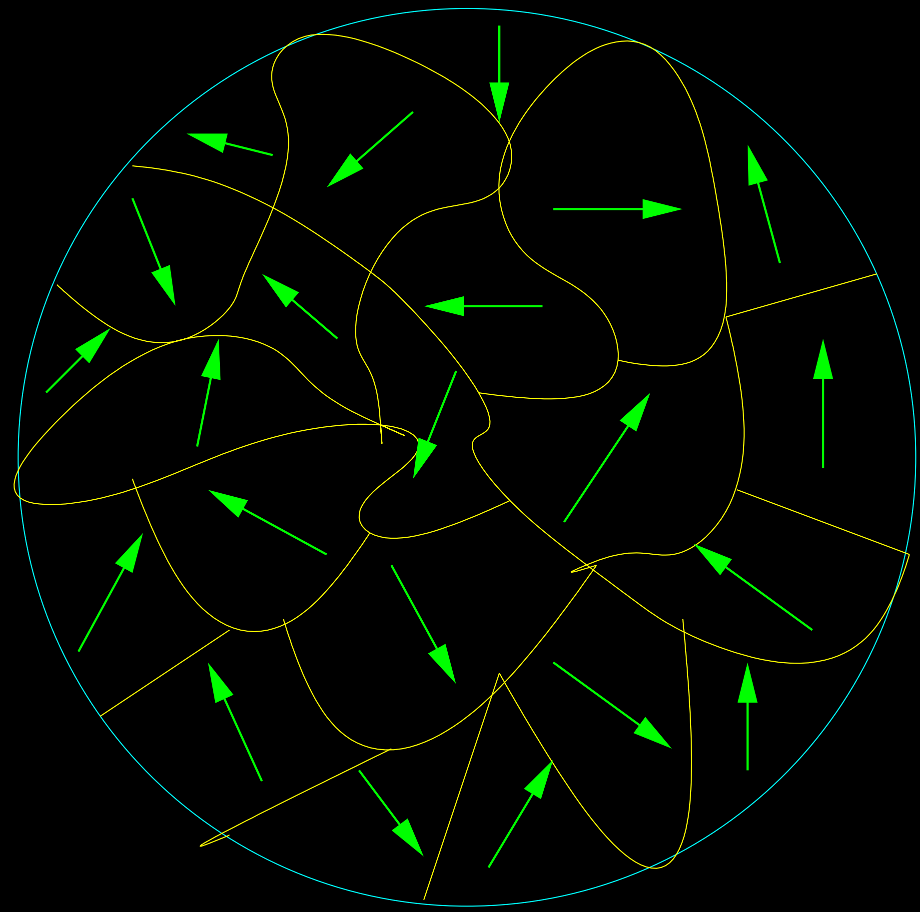
4 dim gravity → 2 dim CFT link

$\Rightarrow$ (Kaul, PM 1998)

$$\begin{aligned} \dim \ \mathcal{H}_{CS+(j_1,...,j_n)} &= \prod_{p=1}^{n} \sum_{m_p=-j_p}^{j_p} [\delta_{m_1+\cdots+m_n,0} \\ &- \frac{1}{2}\delta_{m_1+\cdots+m_n,-1} \\ &- \frac{1}{2}\delta_{m_1+\cdots+m_n,1}] \end{aligned}$$

Term 1 : $m_{tot} = 0 \rightarrow$ overcounting since $j_{tot} = 1, 2, ..$ also have $m_{tot} = 0$; terms 2,3 subtract $m_{tot} = \pm 1$ states

If $j_p = \frac{1}{2} \, \forall \, p = 1, \ldots, n$: **IT from BIT**



Plaquettes have $A_{pl} \sim l_{Pl}^2 : A_{Ibh}/A_{pl} \equiv N_{Ibh} >> 1$

Each Plaq has a binary BIT (e.g., spin 1/2 state) $\Rightarrow$ count total $\dim\{net\ spin = 0\ states\} \equiv \mathcal{N}$

$$\mathcal{N} = \frac{N_{Ibh}!}{((N_{Ibh}/2)!)^2} - \frac{N_{Ibh}!}{(N_{Ibh}/2+1)!(N_{Ibh}/2-1)!}$$

Use Stirling approximation for $N_{Ibh} >> 1$ and $S_{Ibh} \equiv \log \mathcal{N}$ with units chosen such that $k_B = 1$

$$S_{mc} = S_{IH} = \underbrace{\frac{A_{IH}}{4l_P^2}}_{\text{(Ashtekar et. al. 1997)}} \underbrace{- \frac{3}{2}\log\left(\frac{A_{IH}}{4l_P^2}\right) + const. + O(A_{IH}^{-1})}_{\text{(Kaul,PM 2000)}}$$

- **Infinite series of finite, calculable corrections to semicl BHAL : characteristic signature of LQG**
- **Tightening of Bekenstein bound on maximal entropy** Das, Kaul, PM 2001
- **Modified Hawking temperature** $\beta = \beta_H(1 - 6l_P^2/A)$ Majhi, PM 2011

Speculation : quantum origin of Chandrasekhar bound
Chandrasekhar's Nobel Lecture December 1983 : (adapted to deg neutron cores)

Hydrost equil between P_{core} due to gravity and P_{deg} the Fermi pressure of relativistic degenerate neutrons

$$M_{core} \; > \; \xi \; \left(\frac{\hbar c}{G}\right)^{3/2} \; m_n^{-2}$$

Reexpress

$$\left(\frac{M_{core}}{M_P}\right) > \xi \; \left(\frac{\lambda_{Cn}}{l_P}\right)^2$$

Planck scale l_P appears nonperturbatively : $rhs \; \nearrow$ **as** $l_P \searrow$

Reminiscent of black hole entropy :

$$S_{bh} = \frac{A_{hor}}{4 l_P^2} + \text{quantum corr.}$$

- **Is the mass bound linked to quantum gravity ?** Derivation uses GR + Sp Rel QM
- **Are the mass bound and S_{bh} related ?**

Does derivation use a consistent formalism ? No.

- Sp Rel QM not ok for $\mathcal{E} >> m_n c^2 \rightarrow$ SRQFT
- But P_{core} computed using GR : consistency $\Rightarrow$ use GRQFT (semicl) to compute P_{deg}!
- Are QG effects guaranteed to be small ? No

Right answer using 'invalid' theory:
E.g. Mitchell's (1784) derivation of Schwarzschild rad $R_S = 2GM/c^2$ before GR; or Bohr's derivation of Bohr radius $a_0 = \hbar^2/me^2$ before QM.

'Pointers' to the right theory : GR and QM.

What theory does Chandrasekhar's bound point towards ?

Reexpress bound

$$\left(\frac{M_{core}}{M_P}\right) > \xi \,\left(\frac{\lambda_{Cn}}{l_P}\right)^2 = \xi \left(\frac{A_{Cn}}{A_P}\right)$$

$\Rightarrow$ cond for instability wrt formation of **horizon** (spacelike/null trapping hypersurface)

Suggest : existence of bound related to *Stability of horizon wrt Hawking radiation (Thermal Stability)*

$$\frac{M_{hor}(A_{hor})}{M_P} > \frac{S(A_{hor})}{k_B}$$

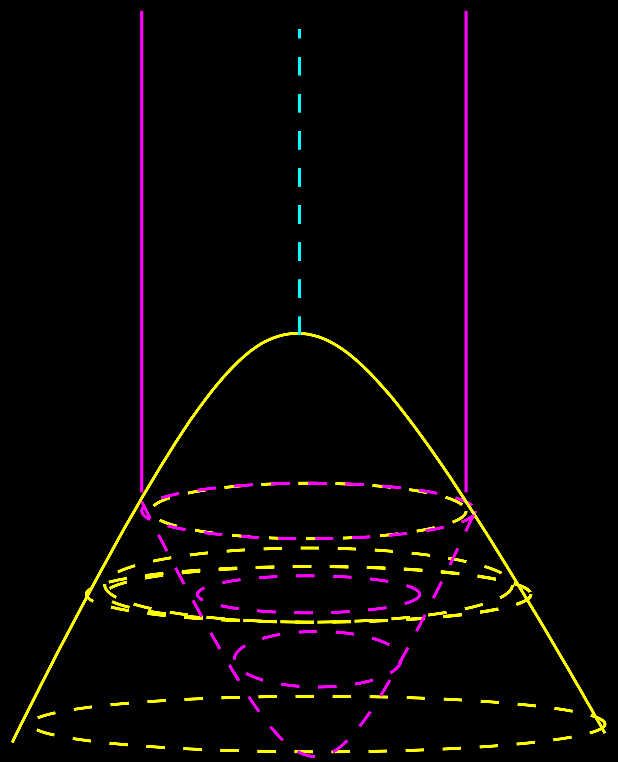
where $S(A_{hor}) \rightarrow$ microcan entropy of equil (isolated) hor

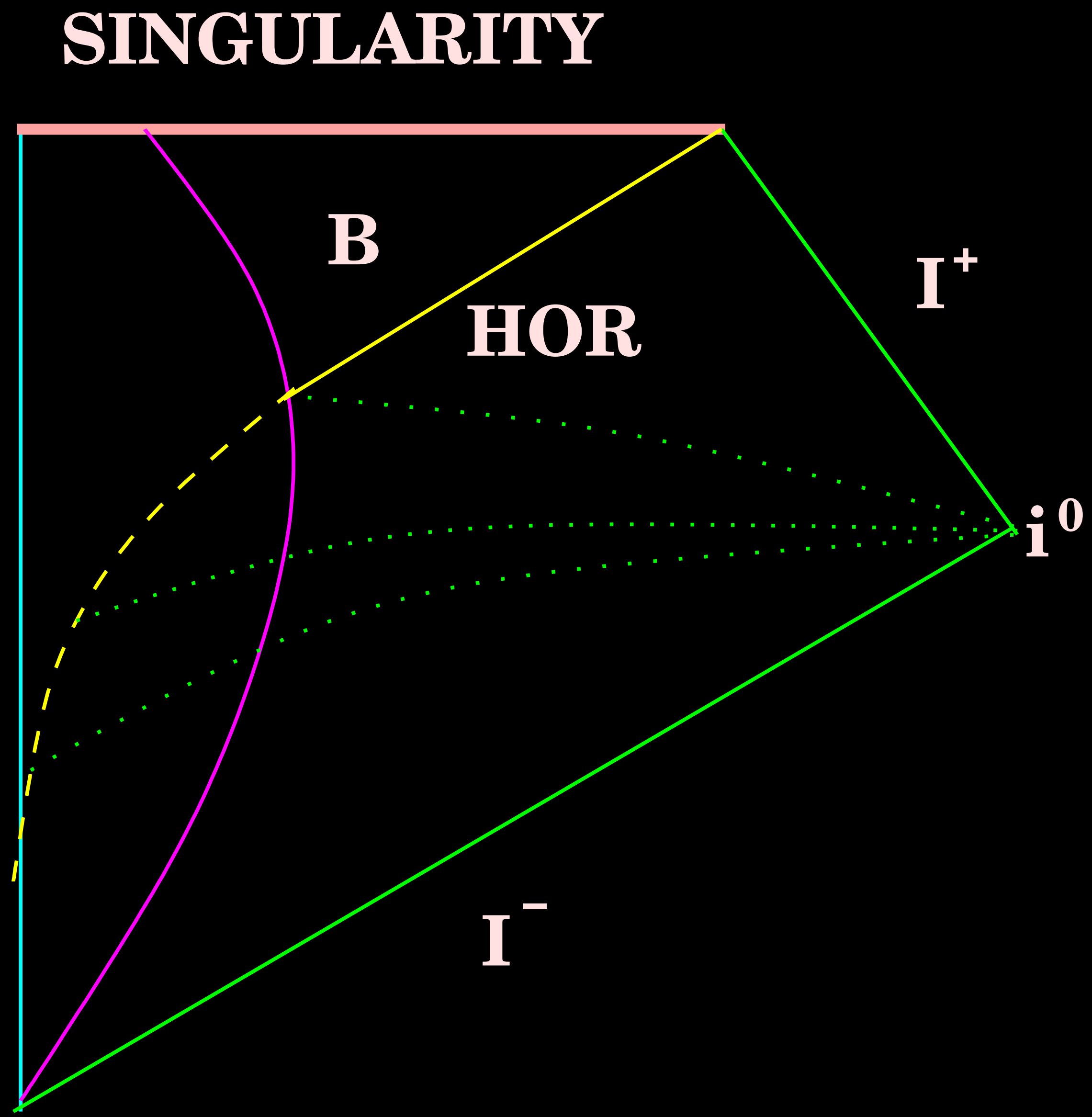
Assume small energy loss during collapse to black hole

$$\Rightarrow M_{core} > M_{hor} = M(A_{hor}) \Rightarrow$$

$$\frac{M_{core}}{M_P} > \frac{A_{hor}}{4l_P^2}$$

Hidden ('Trapping' or Dynamical) horizon of collapsing core $\longrightarrow$ dynamical hypersurface inside core s.t. spatial foliation is outer trapping

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Core Collapse pushes energy into Hidden Horizon $\Rightarrow A_{hid\ hor} \nearrow$

Stops when $A_{hid\ hor} \nearrow A_{hor} \Rightarrow A_{hor} > A_{hid\ hor}$

Expect $A_{Cn} \sim A_{hid\ hor} < A_{hor}$ (?)
$\Rightarrow$ **Chandrasekhar mass bound**

$$\frac{M_{core}}{M_P} > \xi \left(\frac{A_{Cn}}{A_P} \right)$$

Does such a hypersurface actually form in stellar collapse ? Yes, e.g., in Oppenheimer-Snyder model of pressureless dust collapse

Summary

- Weaker version of holography derived from QGR, albeit heuristic
- Can bh entropy receives positive log (area) corrections due to thermal fluct
- Thermal stability: prelim non-semicl understanding why some black holes decay and others may not
- Microcan bh entropy understood for macro bhs; BH area law receives infinite series of finite corrections – signature of LQG
- Bekenstein entropy bound tightened due to LQG corrections
- Possible connection with origin of Chandrasekhar mass bound

Pending Issues

- IH $\longrightarrow$ Dynamical Hor unclear: Hawking radiation ?
- Info Loss Puzzle: can lowest area quantum be a remnant ? Even so, how do we get back lost info ?
- How does LQG resolve black hole singularities ?
- Gauge-gravity connection : relation between Chern Simons dynamics ?
- Detailed check of speculated origin of Chandrasekhar bound