Double Sigma Models and Double Field Theory

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Overview

T-duality is an important property of strings that doesn't exist for point particles: String theory on a circle of radius R is equivalent to string theory on a circle of radius $1/R$.

The (quantised) momentum modes are exchanged with winding around the circle. Splitting the string co-ordinate $X = X_L + X_R$ then the duality replaces it with $\tilde{X} = X_L - X_R$.

For a string theory on a *d*-dimensional torus, the T-duality group is enlarged to O(d,d). There have been many attempts to make this symmetry manifest in the action, usually involving a doubling of co-ordinates to include those dual to winding, like $\tilde{X}$, and this always comes at a price.

Here we seek to connect worldsheet and field theory pictures.

Plan

- The doubled formalism
    - Chirality constraint and integration into action
- The background field method
- Double field theory and generalised Ricci tensor
- Agreement on a ‘fibred’ background
- A more general double sigma model

A duality-invariant picture

- Look for O(d,d) invariance and and new structures which emerge
- A more unified picture of g, b and ϕ
- Doubled geometry and differential geometry
- Geometric description of T-folds; string backgrounds where transition functions can be T-dualities - new compactifications
- String field theoretic motivation for double field theory, $\tilde{X}$ dependent vertex operators - truly doubled theories

The Doubled Formalism

- A sigma model describing a torus fibration in which the fibre co-ordinates are doubled [Hull], $\mathbb{X}^A = (X^i, \tilde{X}_i)$.

- Various other earlier works on doubled sigma models [Tseytlin, Maharana, Schwarz, Sen, Duff,...]

- Minimal Lagrangian: $\mathcal{L} = \frac{1}{4}\mathcal{H}_{AB} d\mathbb{X}^A \wedge * d\mathbb{X}^B + \mathcal{L}(Y)$

- Generalised metric and O(d,d) invariant metric:

$$\mathcal{H}_{AB}(Y) = \begin{pmatrix} h^{-1} & -h^{-1}b \\ bh^{-1} & h - bh^{-1}b \end{pmatrix}, \qquad L_{AB} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

$$\mathcal{H}^{-1} = L^{-1}\mathcal{H}L^{-1}$$

The Constraint

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- We have doubled the number of co-ordinates, if we want to describe the same original string theory we need something else, a constraint which halves the degrees of freedom

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- Explicitly in the simplest case (circle of radius R)

$$P = RX + R^{-1}\tilde{X}, \; \partial_- P = 0\,,$$

$$Q = RX - R^{-1}\tilde{X}, \; \partial_+ Q = 0\,.$$

Incorporating the constraint

- At the classical level the action + constraint give the ordinary string equations of motion. To check quantum equivalence we first incorporate the constraint into the action [Berman, NBC, Thompson].

- We first go to the chiral frame: there we can impose the chirality constraint *a la* PST.

- Written in terms of the chiral P and Q the action has the form

$$S_d = \frac{1}{8}\int dP \wedge *dP + \frac{1}{8}\int dQ \wedge *dQ\,.$$

- We also define vanishing one-forms

$$\mathcal{P} = dP - *dP,\ \ \mathcal{Q} = dQ + *dQ\,.$$

The modified action

- We introduce two closed one-forms to the action

$$S_{PST} = \frac{1}{8}\int dP\wedge *dP + \frac{1}{8}\int dQ\wedge *dQ - \frac{1}{8}\int d^2\sigma\left(\frac{(\mathcal{P}_m u^m)^2}{u^2} + \frac{(\mathcal{Q}_m v^m)^2}{v^2}\right)$$

- Simplest way to proceed is to fix them to be time like. The resulting action is loses manifest Lorentz invariance on the worldsheet

$$\begin{aligned} S &= \frac{1}{4}\int d^2\sigma(\partial_1 P\partial_- P - \partial_1 Q\partial_+ Q). \\ &= \frac{1}{2}\int d^2\sigma\left[-(R\partial_1 X)^2 - (R^{-1}\partial_1\tilde{X})^2 + 2\partial_0 X\partial_1\tilde{X}\right]. \end{aligned}$$

- In the more general case the action takes the following simple form on the fibre, with the base remaining the same

$$\mathcal{L}_{fib} = -\mathcal{H}_{AB}\partial_1\mathbb{X}^A\partial_1\mathbb{X}^B + L_{AB}\partial_0\mathbb{X}^A\partial_1\mathbb{X}^B$$

- The equation of motion integrates to give the constraint.

Background Field Method

- For the classical Weyl invariance of the string to extend to the quantum theory the beta functional must vanish.

- This can be calculated by expanding a quantum fluctuation around a classical background $X^\alpha = X^\alpha_{cl} + \pi^\alpha$ [Honercamp;Alvarez-Gaume, Freedman, Mukhi].

- As π^α does not transform covariantly, one does a more refined expansion to maintain the covariance of the action. ξ^α is the tangent vector to the geodesic from X^α_{cl} to $X^\alpha_{cl} + \pi^\alpha$ with length equal to that of the geodesic.

- The fluctuation propagator can then be obtained and the fluctuations Wick contracted out.

Algorithmic Expansion

- Thanks to [Mukhi] we know a simple algorithmic method to background field expand, simply acting on the Lagrangian n times with the operator
$$\int d^2\sigma \xi^\alpha(\underline{\sigma}) D^\sigma_\alpha$$
- The action is given by

$$\int d^2\sigma\, \xi^\alpha(\underline{\sigma}) D^\sigma_\alpha \xi^\beta(\underline{\sigma}') = 0\,,$$
$$\int d^2\sigma\, \xi^\alpha(\underline{\sigma}) D^\sigma_\alpha \partial_\mu X^\beta(\underline{\sigma}') = D_\mu \xi^\beta(\underline{\sigma}')\,,$$
$$\int d^2\sigma\, \xi^\alpha(\underline{\sigma}) D^\sigma_\alpha D_\mu \xi^\beta(\underline{\sigma}') = R^\beta{}_{\alpha\gamma\delta} \partial_\mu X^\delta \xi^\alpha \xi^\gamma(\underline{\sigma}')\,,$$
$$\int d^2\sigma\, \xi^\alpha(\underline{\sigma}) D^\sigma_\alpha T_{\alpha_1\alpha_2\ldots\alpha_n}(X(\underline{\sigma}')) = D_\beta T_{\alpha_1\alpha_2\ldots\alpha_n} \xi^\beta(\underline{\sigma}')\,,$$

Expansion and propagators

- At second order the result is

$$
\begin{aligned}
2\mathcal{L}_{(2)} =& -\mathcal{G}_{\alpha\beta} D_1\xi^\alpha D_1\xi^\beta + \mathcal{L}_{\alpha\beta} D_0\xi^\alpha D_1\xi^\beta + \mathcal{K}_{\alpha\beta} D_0\xi^\alpha D_0\xi^\beta \\
& - R_{\gamma\alpha\beta\delta}\xi^\alpha\xi^\beta\partial_1 X^\gamma\partial_1 X^\delta + \mathcal{L}_{\alpha\beta;\gamma}\xi^\gamma(D_0\xi^\alpha\partial_1 X^\beta + \partial_0 X^\alpha D_1\xi^\beta) \\
& + \frac{1}{2} D_\alpha D_\beta \mathcal{L}_{\gamma\delta}\xi^\alpha\xi^\beta\partial_0 X^\gamma\partial_1 X^\delta + \frac{1}{2}\left(L_{\gamma\sigma}R^\sigma{}_{\alpha\beta\delta} + L_{\delta\sigma}R^\sigma{}_{\alpha\beta\gamma}\right)\xi^\alpha\xi^\beta\partial_0 X^\gamma\partial_1 X^\delta \\
& + 2\mathcal{K}_{\alpha\beta;\gamma}\xi^\gamma D_0\xi^\alpha\partial_0 X^\beta \\
& + \frac{1}{2} D_\alpha D_\beta \mathcal{K}_{\gamma\delta}\xi^\alpha\xi^\beta\partial_0 X^\gamma\partial_0 X^\delta + \mathcal{K}_{\gamma\sigma}R^\sigma{}_{\alpha\beta\delta}\xi^\alpha\xi^\beta\partial_0 X^\gamma\partial_0 X^\delta
\end{aligned}
$$

- From the kinetic terms in the chiral frame the contractions can be determined to be

$$
\begin{aligned}
\langle\xi^\alpha(z)\xi^\beta(z)\rangle &= \Delta_0\mathcal{G}^{\alpha\beta} + \theta\mathcal{L}^{\alpha\beta}, \\
\langle\xi^\gamma\partial_1\xi^\alpha\partial_1\xi^\rho\xi^\tau\rangle &= -\Delta_0\left(\mathcal{G}^{\alpha[\tau}\mathcal{G}^{\rho]\gamma} - \mathcal{L}^{\alpha[\tau}\mathcal{L}^{\rho]\gamma}\right), \\
\langle\xi^\gamma\partial_1\xi^\alpha\partial_0\xi^\rho\xi^\tau\rangle &= \Delta_0\left(\mathcal{G}^{\alpha[\tau}\mathcal{L}^{\rho]\gamma} + \mathcal{L}^{\alpha[\tau}\mathcal{G}^{\rho]\gamma}\right) + 2\theta\mathcal{L}^{\alpha[\tau}\mathcal{L}^{\rho]\gamma}, \\
\langle\xi^\gamma\partial_0\xi^\alpha\partial_0\xi^\rho\xi^\tau\rangle &= \Delta_0\left(\mathcal{G}^{\alpha[\tau}\mathcal{G}^{\rho]\gamma} + 3\mathcal{L}^{\alpha[\tau}\mathcal{L}^{\rho]\gamma}\right) + 2\theta\left(\mathcal{G}^{\alpha[\tau}\mathcal{L}^{\rho]\gamma} + \mathcal{L}^{\alpha[\tau}\mathcal{G}^{\rho]\gamma}\right).
\end{aligned}
$$

Results

- After much manipulation we are left with the following divergent terms

$$S_{Weyl} = \frac{1}{2}\int d^2\sigma\left[-W_{GD}\partial_1\mathbb{X}^G\partial_1\mathbb{X}^D + W_{gd}\partial_\mu Y^g\partial^\mu Y^d\right]\Delta_0$$

$$W_{GD} = \frac{1}{2}\partial^2\mathcal{H}_{GD} - \frac{1}{2}\left((\partial_a\mathcal{H})\mathcal{H}^{-1}(\partial^a\mathcal{H})\right)_{GD} - \frac{1}{2}\Gamma^t{}_{ab}g^{ab}\partial_t\mathcal{H}_{GD}$$

$$W_{gd} = -\hat{R}_{gd} - \frac{1}{8}\partial_g\mathcal{H}_{AB}\partial_d\mathcal{H}^{AB}$$

- The terms proportional to θ vanish showing Lorentz invariance is maintained.

- After regularising and renormalising the beta functionals vanish if W does. W is not the Ricci tensor of $\mathcal{H}$. More work shows the vanishing of W is the same as the h and b beta functional equations of the undoubled string.

Doubled beta functional

Beta-functional

Ordinary String $\xrightarrow{\text{BFE}}$ $R_{AB} = 0$

Background field equation of string

Doubled beta functional

Beta-functional

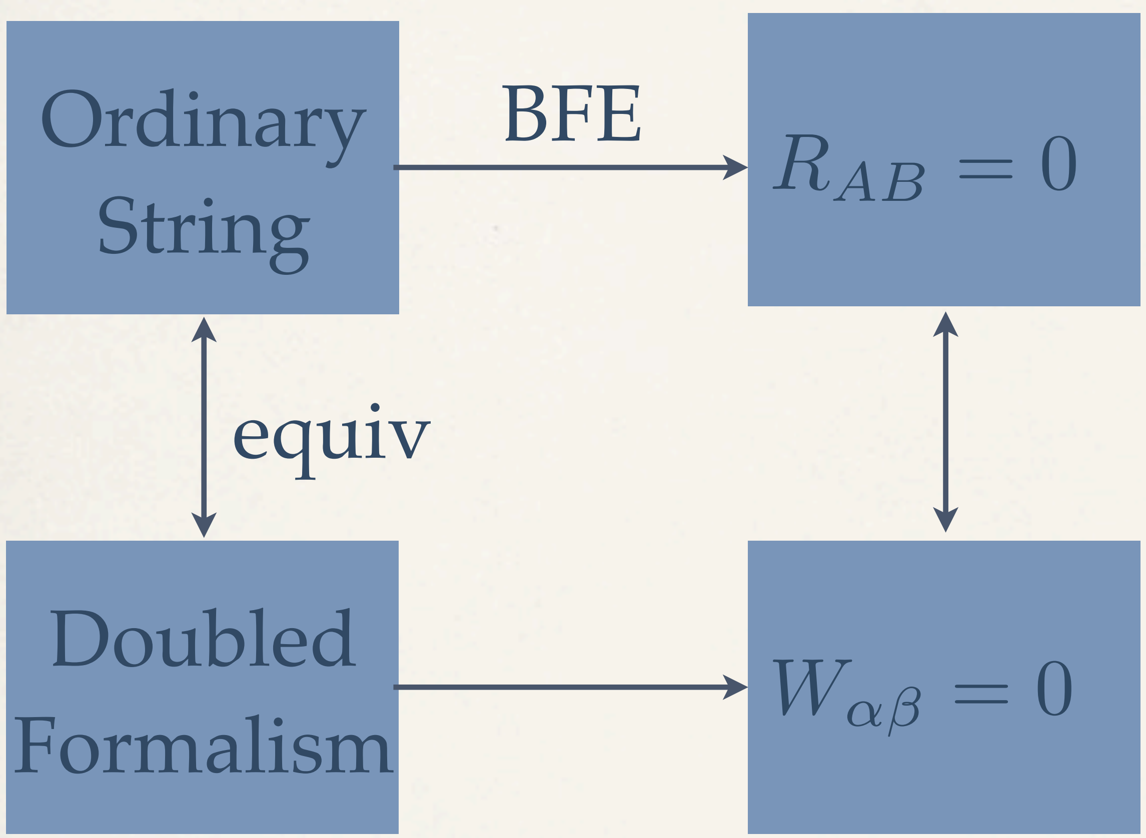
$$\begin{tikzcd}
\text{Ordinary String} \arrow[r, "\text{BFE}"] \arrow[d, leftrightarrow, "\text{equiv}"] & R_{AB} = 0 \arrow[d, leftrightarrow] \\
\text{Doubled Formalism} \arrow[r] & W_{\alpha\beta} = 0
\end{tikzcd}$$

Background field equation of string

Doubled version includes B

Doubled beta functional

Beta-functional



Background field equation of string

Doubled version includes B

The doubled formalism calculation reproduces the string background field equations, including b and dilaton, after a lot of work [BCT].

The ordinary string background field equations can be obtained as equations of motion of an certain action: The string effective action.

Double Field Theory

- Double field theory is a closed string field theory inspired field theory where the fields depend on a doubled set of co-odinates [Hull, Zwiebach, Hohm].

- Fields h, b, ϕ. `Level matching' constraint $\Delta = \partial^M \partial_M = \partial^i \tilde{\partial}_i$ acts on fields. Originally a fully doubled theory, action found to third order in perturbations.

- Imposing strong version of the constraint, that Δ annihilate any product of fields, means we can O(d,d) rotate to frame where they depend only on X^i. Background independent action can be written in terms of

$$\mathcal{E} = h + b$$

$$e^{-2d} = \sqrt{h} e^{-2\phi}$$

- $\mathcal{E}$ transforms non-linearly under O(d,d). And in a complicated fashion under a double gauge-symmetry.

Generalised Metric Formulation

- In this restricted case can reformulate in terms of the familiar looking generalised metric which transforms linearly under O(d,d)

$$\mathcal{H}_{MN} \;=\; \left(\begin{array}{cc} h^{-1} & -h^{-1}b \\ bh^{-1} & h-bh^{-1}b \end{array}\right).$$

- In fact the action can even be written in Einstein-Hilbert form for a gauge scalar $\mathcal{R}$

$$S=\int dx d\tilde{x} e^{-2d}\mathcal{R}$$

- The equation of motion can be written in terms of a `generalised Ricci tensor'

$$\mathcal{R}_{MN}=\frac{1}{2}\left(\mathcal{K}_{MN}-\mathcal{H}_M^{\;P}\mathcal{K}_{PQ}\mathcal{H}^Q_{\;\;N}\right)$$

$$\begin{aligned}\mathcal{K}_{MN} \;=\;& \frac{1}{8}\,\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL}-\frac{1}{4}(\partial_L-2(\partial_L d))(\mathcal{H}^{LK}\partial_K\mathcal{H}_{MN})+2\,\partial_M\partial_N d\\ &-\frac{1}{2}\partial_{(M}\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{N)K}+\frac{1}{2}(\partial_L-2(\partial_L d))\big(\mathcal{H}^{KL}\partial_{(M}\mathcal{H}_{N)K}+\mathcal{H}^K{}_{(M}\partial_K\mathcal{H}^L{}_{N)}\end{aligned}$$

Double gauge-transform

- The double gauge transform is an O(d,d) form of diffeomorphism and gauge transformation. It acts on the generalised metric in the like a modified diffeomorphism

$$\begin{aligned}\delta_\xi \mathcal{H}^{MN} &= \xi^P \partial_P \mathcal{H}^{MN} + (\partial^M \xi_P - \partial_P \xi^M)\,\mathcal{H}^{PN} + (\partial^N \xi_P - \partial_P \xi^N)\,\mathcal{H}^{MP} \\ &= \widehat{\mathcal{L}}_\xi \mathcal{H}^{MN}\end{aligned}$$

- The double gauge transform's algebra is also a generalisation of the Lie derivatives

$$\left[\,\widehat{\mathcal{L}}_{\xi_1}\,,\widehat{\mathcal{L}}_{\xi_2}\,\right] = -\widehat{\mathcal{L}}_{[\xi_1,\xi_2]_{\mathrm{C}}}$$

- The C-bracket is an extension of the Courant bracket. The appearance of the generalised metric and Courant bracket is reminiscent of Generalised geometry [Hitchin, Gualtieri; Waldram *et al*].

The generalised Ricci tensor

- We identified $\mathcal{R}_{MN}$ as the generalised Ricci tensor. It contains b and ϕ as well as g . If it is an analogue of the Ricci tensor then it is on some new O(d,d) differential geometry.

- Various approaches. [Park, Jeon, Lee] work in terms of the projector

$$P_{AB} = (L_{AB} + \mathcal{H}_{AB})/2 \qquad\qquad P_A{}^B P_B{}^C = P_A{}^C$$

- They define a "semi-covariant" derivative which annihilates all the fields: L_{AB}, P_{AB}, d . The generalised Ricci tensor can be described in terms of this and the projectors.

- Other approaches: Vielbeins [Siegel, Hohm, Kwak], generalised geometry [Waldram et al], but interrelated.

Doubled field theory extensions...

- Fermions, supergravity: [Hohm, Kwak, Park,....]
- Branes: [Bergshoeff, Riccioni, Albertsson *et al*,...]
- M-Theory generalised geometry: [Berman, Perry, Godazgar]. Reduction to double field theory giving RR fields [Thompson].
- Doubled Heterotic, Doubled Yang-Mills, Doubled KK monopoles... doubles all the way.
- A versatile framework that seems to have wider applicability.

Reduction of the 'Ricci Tensor'

The doubled field theory is more generally defined. To connect the two theories we must restrict to the fibred background of the doubled formalism [NBC; 1106.1888]. We must

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- Undouble the base co-ordinate $\dfrac{\partial}{\partial \tilde{y}_a} = 0$
- No b on the base and define the correct semi-doubled dilaton which takes into account only the doubling of the fibre.

$$\partial_A d = -\frac{1}{4} g^{ab} \partial_A g_{ab} + \partial_A \Phi$$

Dilaton

$$\partial_A d = -\frac{1}{4} g^{ab} \partial_A g_{ab} + \partial_A \Phi$$

- The so far the discussion of the doubled formalism didn't contain the dilaton, but we note that the dilaton terms in the generalised Ricci tensor can be written as proportional to our equation of motion

$$-\frac{1}{2} g^{kl} \partial^p g_{kl} \left(\hat{D}_\mu (g_{pn} \partial^\mu X^n) - \frac{1}{2} \partial_p \mathcal{H}_{MN} \partial_1 X^M \partial_1 X^N \right)$$

- The Φ parts can be rewritten $\mathcal{R}_{\Phi\ \mu\nu} = D_\mu D_\nu \Phi$

- We recognise this as the shift in the beta-function by introducing a particular counter term - the Fradikn-Tseytlin term for the Dilaton.

The connection is made...

- We find $W_{\alpha\beta} = -\frac{1}{2}\mathcal{R}_{\alpha\beta}$ so that indeed the vanishing of the beta functional in the doubled formalism is the same as the equation of motion of the double field theory in this restricted set up.

- This had to happen as both theories should be equivalent to the undoubled equivalents. The important thing is the central role of $\mathcal{R}_{\alpha\beta}$ which plays the role of a doubled Ricci tensor (NB it is not the Ricci tensor of $\mathcal{H}_{\alpha\beta}$), it contains b and the dilaton.

- Can it be interpreted within a more general doubled differential geometry? [Holm & Kwak; Jeon, Lee & Park, Waldram et al]

A more general sigma model

- One is lead to ask the question if a more general sigma model exists, that gives the full generalised Ricci tensor as its background field equation?

- There are technical and conceptual difficulties. Although toroidal directions are needed for the O(d,d) rotations to describe a T-duality, the formalism can be used more generally, as in the double field theory case.

- We propose[NBC:1111:1828] that the following action leads to double field theory:
$$S = \frac{1}{2}\int d^2\sigma \left[-\mathcal{H}(X)_{MN}\partial_1 X^M \partial_1 X^N + L_{MN}\partial_1 X^M \partial_0 X^N\right]$$

- General actions of this type were studied by [Tseytlin], the restriction to the geometry to a group manifold was examined by [Avramis, Derendinger & Prezas; Sfetsos, Siampos & Thompson]

Constraint again.

- A key point is although $\mathcal{H}$ can depend on the doubled co-ordinates, this dependence is not arbitrary, we impose the level matching constraint as in double field theory.

- Of course this means that we can rotate to a frame where there is no dependence on the dual co-ordinates things should be equivalent to the ordinary string sigma model.

$$\frac{\partial}{\partial \tilde{X}} = 0$$

- The strength of the strong constraint, restricting the co-ordinate dependence of all fields to an isotropic subspace means that it holds even if the fields are evaluated at different points

$$\partial_M A(X(\sigma))\partial^M B(X(\sigma')) = 0$$

Equation of motion

- The classical equation of motion is

$$\partial_1(\mathcal{H}_{MN}\partial_1 X^B - L_{MN}\partial_0 X^N) = \frac{1}{2}\partial_M \mathcal{H}_{NP}\partial_1 X^N \partial_1 X^P$$

- It is no longer a total derivative, but we integrate anyway

$$\mathcal{H}_{MN}\partial_1 X^N - L_{MN}\partial_0 X^N = \frac{1}{2}\int d\sigma_1' \epsilon(\sigma_1 - \sigma_1')[\partial_M \mathcal{H}_{NP}\partial_1 X^N \partial_1 X^P](\sigma')$$

- The LHS is just the constraint of the doubled formalism in our case. We find in working with the general sigma model we would like to use this constraint. What about the non-local term?

- It turns out when the constraint is needed it is always contracted with a derivative ∂^M, so the non-local term does not contribute.

- This is clearly seen in the canonical duality frame.

Classical equivalence

- We can then check classical Lorentz invariance by introducing a world sheet vielbein. The condition is basically the vanishing of the chirality constraint squared contracted with L.
- We can check equivalence of the equations of motion to the ordinary string sigma model.
- In both cases the proof relies on being able to integrate half of the components of the equation of motion, and these being the only components we need.

Doubled gauge transformations

- The first term in the action is invariant if $\partial_1 X^M = \begin{pmatrix} \partial_1 \tilde{X}_i \\ \partial_1 X^i \end{pmatrix}$ transforms correctly under double gauge transforms.

- We know how X^i transforms, and we get the transformation of $\partial_1 \tilde{X}_i$ through the components of the equation of motion we know, which state

$$\partial_1 \tilde{X}_i = g_{ij} \partial_0 X^j + b_{ij} \partial_1 X^j$$

- We know how the undoubled fields transform and we get the right transformation

$$\begin{aligned} \delta_\xi (\partial_1 \tilde{X}_i) =& (\xi^k \partial_k g_{ij} + \partial_i \xi^k g_{kj}) \partial_0 X^j + (\xi^k \partial_k b_{ij} + \partial_i \xi^k b_{kj}) \partial_1 X^j \\ & + (\partial_i \tilde{\xi}_j - \partial_j \tilde{\xi}_i) \partial_1 X^j \\ =& \xi^k \partial_k \partial_1 \tilde{X}_i + \partial_i \xi^k \partial_1 \tilde{X}_k + (\partial_i \tilde{\xi}_j - \partial_j \tilde{\xi}_i) \partial_1 \tilde{X}^j \,, \end{aligned}$$

- However, the L term needs something extra.

The Topological term

- The L term contains $\partial_0 \tilde{X}_i$, but that half of the equation of motion cannot be integrated.

- We can remove $\partial_0 \tilde{X}_i$ from the action by adding a total derivative. The new term has the correct gauge transform, but the difference of its transport term from that of the original term is not a total derivative - the double gauge invariant action includes the total derivative which can be written
$$\mathcal{L}_{top} = \frac{1}{2}\Omega_{MN}\partial_1 X^M \partial_0 X^N \qquad \Omega_{MN} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- Such a topological term was also needed in the doubled formalism to ensure gauge invariance under large gauge transformations [Hull] and was needed in showing equivalence of the doubled string partition function to its ordinary counterpart [Berman &NBC].

- Is not manifestly O(d,d) invariant, but plays no role in what follows.

Background field expansion

- The background field expansion proceeds as before. We need to use the integrated half of the equation of motion to eliminate (for instance) terms proportional to $\partial_0 X^M \partial_0 X^N$.
- Lorentz invariance at one loop is also demonstrated.
- Dilaton terms can also be included as they vanish on shell after use of the equation of motion.
- The result is the background field equation is proportional to generalised Ricci tensor of doubled field theory!
- Recall this indicates that the (restricted) double field theory is the effective field theory for the more general sigma model.

Other questions

- If the double field theory is the effective field theory for the sigma model, then we should be able to find higher-order corrections by doing the background field expansion to higher order.
- Two-loop calculation underway: many complications, expect $\mathcal{H}$ to have α' corrections (see [Meissner], [Hohm&Zwiebach]).
- Perhaps this would be easier if the expansion was done in a derivative more suited to the double geometry.
- Is there a Lorentz invariant (plus constraint) Lagrangian on which the PST procedure can be performed to get our Lagrangian. Only half of the constraint must be imposed?
- Can we relax the strong constraint (c.f. compactification of DFT to give gauged supergravities.Truly doubled theory?

Conclusion

- The doubled formalism provides a T-duality symmetric sigma model for a certain class of fibred backgrounds. It aims to make non-geometric backgrounds such as T-folds geometric.
- Double field theory hopes to describe a truly doubled field theory, but in restricted generalised metric formulation brings new doubled geometric structures to the fore.
- It we restrict double field theory to the kind of background to which the doubled formalism applies, the equation of motion of the former is the background field equation of the latter.
- We can go further: a more general sigma model with metric dependent on all the doubled directions gives the full equation of motion of doubled field theory as its background field equation.

Thank you!