• Rigid SUSY, conformal coupling and twistor spinors •

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Some context and motivation

Even after almost half a century, rather remarkable how SUSY continues to elucidate many fundamental aspects of quantum field theory.

SUSY representations may be classed as either ~~global~~ rigid or local, according to how the SUSY parameter depends on the background geometry.

Often possible to couple a rigid supermultiplet in flat space to supergravity such that it retains local SUSY in curved space (e.g. induced holographically).

Curved backgrounds which support rigid SUSY are more discriminating.

Some recent interest in exploring beyond the few known examples toward a more systematic characterisation of rigid supermultiplets in curved space.

Motivation stems from the plethora of impressive exact results obtained over the past few years via localisation of path integrals for certain operators in field theories with rigid SUSY on (products and quotients of) spheres.

A few highlights:

- [Pestun(2007)] $\rightsquigarrow$ Wilson and 't Hooft operators in $\mathcal{N}=4$ SYM on S^4.
  SCFT indices on $S^3 \times S^1$ and Seiberg dualities.
- [KWY,(D)MP(2010)] $\rightsquigarrow$ $\mathcal{N} \geq 2$ SCFT partition functions on S^3.
  Exact R-charges in $\mathcal{N}=2$ SCFT and F-theorem.
  $N^{3/2}$ scaling law for N M2-branes.
- [K(Q)Z,HST(2012)] $\rightsquigarrow$ SYM partition function on S^5.
  N^3 scaling law for N M5-branes.

Key ingredient is contribution from non-minimal curvature couplings needed for rigid SUSY in curved space.

(e.g. scalar curvature behaves as infrared regulator in correlation functions.)

Path integral localised on fixed points of rigid SUSY which preserves operators in correlator – typically reduces to exactly solvable matrix model!

State of the art

In principle, Noether procedure should determine if rigid SUSY is possible on a given curved background $\mathscr{M}$.

In practise, this is rather cumbersome and must be applied case by case.

More efficient strategy pioneered in 4d by [Festuccia+Seiberg(2011)]:

– Rigid limit of non-linear σ-model coupled to off-shell Poincaré supergravity.

– Planck mass $\to \infty$ and dynamics of gravity supermultiplet frozen out.

– Gravitino and its SUSY variation set to zero

$\rightsquigarrow$ bosonic supergravity background supporting rigid SUSY.

(Auxiliary fields present and need not solve all supergravity field equations.)

Similar deal for SCFTs coupled to conformal supergravity in 4d.

General picture is that $\mathscr{M}$ supports rigid SUSY with parameter ϵ obeying twistor spinor (a.k.a. CKS) equation $\mathscr{D}_\mu \epsilon = \frac{1}{4} \Gamma_\mu \not{\mathscr{D}} \epsilon$ w.r.t. connection $\mathscr{D}_\mu = \nabla_\mu + i a_\mu \Gamma$ on spinor bundle, for some background one-form a.

Details depend critically on whether metric on $\mathscr{M}$ has lorentzian or euclidean signature since this governs the type of spinors which $\mathscr{M}$ supports and therefore the type of supermultiplets which can exist.

For example, in 4d, when

– $\mathscr{M}$ riemannian $\Rightarrow$ Majorana spinors ✗ $\Rightarrow$ rigid SUSY if $\mathscr{M}$ is hermitian.
– $\mathscr{M}$ lorentzian $\Rightarrow$ Majorana spinors ✓ $\Rightarrow$ rigid SUSY if $\mathscr{M}$ admits CKV.

Since localisation most straightforward on compact manifolds, original focus mainly on riemannian case though the lorentzian case, which is more suited to holographic applications, is now receiving more attention.

Stick with lorentzian signature here and see how compatibility of conformal and spin structure on $\mathscr{M}$ affects formulation of field theories with rigid SUSY.

Look at this in the context of some well-known minimal supermultiplets in dimensions $\mathrm{d} = 3, 4, 6, 10$ to discover some novel couplings with rigid SUSY.

But first, the requisite spinorial yoga

Minkowski space $\mathbb{R}^{1,\mathsf{d}-1}$ with flat metric $\eta_{\mu\nu}$ of mostly plus signature.

Clifford algebra $C\ell(1, \mathsf{d}-1)$ generated by Γ_μ with $\Gamma_\mu\Gamma_\nu + \Gamma_\nu\Gamma_\mu = 2\eta_{\mu\nu}\mathbf{1}$.

– Convenient basis in terms of $\Gamma_{\mu_1\ldots\mu_k} = \Gamma_{[\mu_1}\ldots\Gamma_{\mu_k]}$.

– Lorentz subalgebra $\mathfrak{so}(1, \mathsf{d}-1) < C\ell(1, \mathsf{d}-1)$ spanned by $\frac{1}{2}\Gamma_{\mu\nu}$.

Let $\mathfrak{d} = \lfloor \frac{\mathsf{d}}{2} \rfloor$ and work in $2^{\mathfrak{d}}$-dim 'gamma matrix' irrep of $C\ell(1, \mathsf{d}-1)$.

– Can and will take gamma matrices unitary, i.e. $\Gamma_\mu^\dagger = \Gamma^\mu$.

Dirac spinor rep $\mathfrak{S}$ defined by action of $\mathfrak{so}(1, \mathsf{d}-1)_{\mathbb{C}}$ on $\mathbb{C}^{2^{\mathfrak{d}}}$.

– Dirac conjugate $\psi^\dagger\Gamma^0$ of $\psi \in \mathfrak{S}$ defines a hermitian structure on $\mathfrak{S}$.

Involution $\Gamma_\mu \mapsto \sigma_{\mathbf{X}}\,\mathbf{X}\,\Gamma_\mu\mathbf{X}^{-1}$ for some $\sigma_{\mathbf{X}} = \pm 1$, $\mathbf{X} \in \mathrm{GL}(2^{\mathfrak{d}}, \mathbb{C})$.

– Transposition $\Gamma_\mu \mapsto \Gamma_\mu^t$ with $\mathbf{X} = \mathbf{C}$ charge conjugation matrix.

– Complex conjugation $\Gamma_\mu \mapsto \Gamma_\mu^*$ with $\mathbf{X} = \mathbf{B}$ and $\sigma_{\mathbf{B}} = -\sigma_{\mathbf{C}}$

$\rightsquigarrow$ $\mathbf{B}$ is unitary and $\mathbf{B}^*\mathbf{B} = \sigma_{\mathbf{C}}(-1)^{\mathfrak{d}(\mathfrak{d}+1)/2}\mathbf{1}$.

Reality condition $\psi^* = \mathbf{B}\psi$ defines Majorana spinor rep.

- $\mathbf{B}$ defines a real structure on $\mathfrak{S}$ (requires $\mathbf{B}^*\mathbf{B} = \mathbf{1}$).
- Compatibility condition: Majorana conjugate $\overline{\psi} := \psi^t \mathbf{C} = \psi^\dagger \Gamma^0$.
- Exist in $\mathsf{d} = 2, 3, 4 \bmod 8$ where $\mathrm{C}\ell(1, \mathsf{d}-1)$ is matrix algebra over $\mathbb{R}$.

Reality condition $(\psi^A)^* = \varepsilon_{AB}\mathbf{B}\psi^B$ defines symplectic Majorana spinor rep, where ψ^A transform as $\mathfrak{usp}(2)$-doublet of Dirac spinors.

- $\mathbf{B}$ defines a quaternionic structure on $\mathfrak{S}$ (requires $\mathbf{B}^*\mathbf{B} = -\mathbf{1}$).
- Compatibility condition: $\varepsilon_{AB}\overline{\psi}^B = (\psi^A)^\dagger \Gamma^0$.
- Exist in $\mathsf{d} = 6, 7, 8 \bmod 8$ where $\mathrm{C}\ell(1, \mathsf{d}-1)$ is matrix algebra over $\mathbb{H}$.

For d even, Chirality matrix $\Gamma \in \mathrm{C}\ell(1, \mathsf{d}-1)$ obeys $\Gamma^2 = \mathbf{1}$ and $\Gamma_\mu \Gamma = -\Gamma\Gamma_\mu$.

- Projectors $\mathbf{P}_\pm = \frac{1}{2}(\mathbf{1} \pm \Gamma)$ define $\pm$ chirality Weyl spinor reps $\psi_\pm := \mathbf{P}_\pm \psi$.
- $\Gamma^* = \mathbf{B}\Gamma\mathbf{B}^{-1}$ in $\mathsf{d} = 2 \bmod 4$ while $\Gamma^* = -\mathbf{B}\Gamma\mathbf{B}^{-1}$ in $\mathsf{d} = 4 \bmod 4$.
- Majorana-Weyl in $\mathsf{d} = 2 \bmod 8$, symplectic Majorana-Weyl in $\mathsf{d} = 6 \bmod 8$.

Dirac current $\xi_\mu = \bar{\epsilon}\,\Gamma_\mu \epsilon$ of a non-zero bosonic (symplectic) Majorana spinor ϵ defines a real non-zero vector on $\mathbb{R}^{1,\mathsf{d}-1}$ that is either timelike or null.

Furthermore, ξ is null only if $\not{\xi}\epsilon = 0$ and $\bar{\epsilon}\epsilon = 0$

– this is so in $\mathsf{d} = 3, 4$ and also $\mathsf{d} = 6, 10$ if ϵ is chiral.

[**Note:** Four normed division algebras $\mathbb{D} = \mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$ have $\dim \mathbb{D} + 2 = \mathsf{d}$.]

In precisely these four cases, map $\pi : \epsilon \mapsto \xi$ has interesting structure;

- Space of spinors ϵ with unit norm isomorphic to $S^{2\mathsf{d}-5}$.
- $\pi(S^{2\mathsf{d}-5}) \cong S^{\mathsf{d}-2}$ defines 'celestial' sphere in $\mathbb{R}^{1,\mathsf{d}-1}$ (with ξ^0 fixed).
- Inverse image of a point in $S^{\mathsf{d}-2}$ isomorphic to (parallelisable) $S^{\mathsf{d}-3} \subset \mathbb{D}$.
- Recover four Hopf fibrations $S^{\mathsf{d}-3} \hookrightarrow S^{2\mathsf{d}-5} \to S^{\mathsf{d}-2} \cong \mathbb{D}\mathrm{P}^1$.

Straightforward extension on a general lorentzian spin manifold $\mathscr{M}$, with respect to choice of orthonormal frame bundle which preserves spin structure

– refer to canonical extension as minimal coupling.

G-structure and intrinsic torsion

Assume ϵ is a nowhere-vanishing section of spinor bundle, with stabiliser isomorphic to $H_\epsilon < \mathrm{Spin}(1, \mathsf{d}-1)$ at each point in $\mathscr{M}$.

Defines a so-called G-structure on $\mathscr{M}$, with $\mathsf{G} = H_\epsilon$.

Homogeneous space $\mathrm{Spin}(1, \mathsf{d}-1)/H_\epsilon$ parameterises reductions of structure group of frame bundle from $\mathrm{Spin}(1, \mathsf{d}-1)$ to H_ϵ.

For a given reduction, $\exists$ unique connection w.r.t. which ϵ is parallel
– connection has intrinsic torsion $\tau \in T^*\mathscr{M} \otimes \mathfrak{so}(1, \mathsf{d}-1)/\mathfrak{h}_\epsilon$.

If ξ is null, $\mathfrak{h}_\epsilon \cong \mathfrak{g}_\epsilon \ltimes \mathbb{R}^{\mathsf{d}-2}$, where $\mathfrak{g}_\epsilon < \mathfrak{so}(\mathsf{d}-2)$ isotropy of spinor in $\mathbb{R}^{\mathsf{d}-2}$
– in fact, $\mathfrak{g}_\epsilon \cong \mathfrak{so}(\mathsf{d}-3)$ in $\mathsf{d} = 3, 4, 6, 10$.

If ξ is a null conformal Killing vector then ϵ obeys twistor spinor equation

$$\mathscr{D}_\mu \epsilon = \tfrac{1}{\mathsf{d}} \Gamma_\mu \not{\mathscr{D}} \epsilon$$

w.r.t. $\mathscr{D} = \nabla + \mathsf{t}$, where $\mathsf{t} \in \mathbb{R}^{1,\mathsf{d}-1} \otimes \mathfrak{so}(\mathsf{d}-2)/\mathfrak{g}_\epsilon \oplus \mathfrak{g}_\epsilon$ at a point in $\mathscr{M}$.

[**Note:** $T_e S^{\mathsf{d}-3} \cong \mathfrak{so}(\mathsf{d}-2)/\mathfrak{so}(\mathsf{d}-3) \rightsquigarrow$ R-symmetry action in $\mathsf{d} = 3, 4, 6$.]

Rigid SUSY and conformal coupling

In a field theory on $\mathscr{M}$ with minimal rigid SUSY generated by δ_ϵ, (off-shell) closure of the SUSY algebra means δ_ϵ^2 must generate a bosonic symmetry.

Typically δ_ϵ^2 contains several contributions;

- Lie derivative $\mathcal{L}_\xi$ along Dirac current $\xi^\mu = \bar{\epsilon}\,\Gamma^\mu \epsilon$.
- Homothety δ_σ with parameter $\sigma = -\frac{1}{\mathsf{d}}\nabla_\mu \xi^\mu$.
- R-symmetry variation δ_ρ with parameter ρ.
- Gauge transformation δ_Λ with parameter $\Lambda = -\xi^\mu A_\mu$.
- Equations of motion, for on-shell supermultiplets.

[**Note:** It is the spinorial lie derivative $\mathcal{L}_\xi \psi = \xi^\mu \nabla_\mu \psi + \frac{1}{4}(\nabla_\mu \xi_\nu)\Gamma^{\mu\nu}\psi$ which acts on a spinor ψ when ξ generates a conformal isometry of $\mathscr{M}$.]

Q: Given a classical superconformal field theory in Minkowski space, can it be reformulated as a theory with rigid SUSY on $\mathscr{M}$?

A: Yes, provided $\mathscr{M}$ admits twistor spinors – straightforward reformulation based on conformal coupling of lagrangian and SUSY variations.

Scale-invariant field theory on $\mathbb{R}^{1,\mathsf{d}-1}$ whose fields Φ have dimensions Δ_Φ

$\rightsquigarrow$ Minimally coupled theory on $\mathscr{M}$ has global Weyl symmetry w.r.t. weights $w_\Phi = r_\Phi - \Delta_\Phi$, where r_Φ is tensorial rank of Φ.

However, full Weyl-invariance typically requires additional improvement terms which exist only if original theory conformally invariant on $\mathbb{R}^{1,\mathsf{d}-1}$.

Under Weyl transformation of fields $\Phi \mapsto \Omega^{w_\Phi}\Phi$ and metric $g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}$,

$$\nabla_\mu - \tfrac{1}{\mathsf{d}}\Gamma_\mu \nabla\!\!\!/ \mapsto \Omega^{\frac{1}{2}} \left(\nabla_\mu - \tfrac{1}{\mathsf{d}}\Gamma_\mu \nabla\!\!\!/ \right) \Omega^{-\frac{1}{2}}$$

so defining equation for twistor spinor ϵ is Weyl-invariant with $w_\epsilon = \frac{1}{2}$.

Example 1: free scalar supermultiplet

Need bosonic scalar Φ paired with fermionic spinor Ψ on $\mathbb{R}^{1,\mathsf{d}-1}$.

Match on-shell d.o.f. in $\mathsf{d}=3,4,6$ with reps based on $\mathbb{A}=\mathbb{R},\mathbb{C},\mathbb{H}$ structure.
(Φ is $\mathbb{A}$-valued, Ψ from Majorana, Weyl, symplectic Majorana-Weyl rep.)

On-shell SUSY variations of the form

$$\delta_\epsilon \Phi = \overline{\epsilon}' \Psi \ , \qquad\qquad \delta_\epsilon \Psi = \Gamma^\mu \epsilon \, \partial_\mu \Phi$$

where $\epsilon' = (\mathbf{B}\epsilon)^*$ of same spinorial type as Ψ.

SUSY algebra closes on-shell with $\delta_\epsilon^2 = \xi^\mu \partial_\mu$ using $\epsilon\overline{\epsilon}' = \frac{1}{2}\not{\xi}$ on Ψ and field equation $\not{\partial}\Psi = 0$ (translation paramater $\xi^\mu = \overline{\epsilon}'\Gamma^\mu\epsilon$ is real and null).

SUSY lagrangian of the form

$$\langle \partial_\mu \Phi, \partial^\mu \Phi \rangle + \langle \overline{\Psi}, \not{\partial}\Psi \rangle$$

where $\langle -,- \rangle$ is real part of euclidean inner product on $\mathbb{A}$.

Scale-invariant with $\Delta_\Phi = \frac{\mathsf{d}}{2} - 1$, $\Delta_\Psi = \Delta_\Phi + \frac{1}{2}$ and $\Delta_\epsilon = -\frac{1}{2}$.

Now conformally couple on $\mathscr{M}$:

- On-shell SUSY variations become

$$\delta_\epsilon \Phi = \overline{\epsilon}' \Psi \ , \qquad \delta_\epsilon \Psi = \Gamma^\mu \epsilon \, \nabla_\mu \Phi + \left(1 - \tfrac{2}{\mathsf{d}}\right) \nabla\!\!\!/ \epsilon \, \Phi$$

- Rigid SUSY algebra now closes on-shell with

$$\delta_\epsilon^2 = \mathcal{L}_\xi + \delta_\sigma + \delta_\rho$$

using Weyl-invariant field equation $\nabla\!\!\!/ \Psi = 0$, provided ϵ is a twistor spinor.

– Null conformal Killing vector $\xi^\mu = \overline{\epsilon}' \Gamma^\mu \epsilon$.

– Homothety parameter $\sigma = -\frac{1}{\mathsf{d}} \nabla_\mu \xi^\mu$.

– R-symmetry parameter ρ is $\mathrm{Im}\, \mathbb{A}$-valued, proportional to $\overline{\epsilon}' \nabla\!\!\!/ \epsilon - \mathrm{Re}\, (\overline{\epsilon}' \nabla\!\!\!/ \epsilon)$.

- SUSY lagrangian becomes

$$\langle \nabla_\mu \Phi, \nabla^\mu \Phi \rangle + \tfrac{\mathsf{d}-2}{4(\mathsf{d}-1)} R \langle \Phi, \Phi \rangle + \langle \overline{\Psi}, \nabla\!\!\!/ \Psi \rangle$$

- Can further generalise by gauging R-symmetry:
  $\nabla \rightsquigarrow \mathscr{D} = \nabla + \mathsf{a}$ for any Weyl-invariant, $\mathrm{Im}\, \mathbb{A}$-valued one-form a on $\mathscr{M}$.

Example 2: Yang-Mills supermultiplet

Need gauge field A_μ paired with gaugino λ on $\mathbb{R}^{1,\mathsf{d}-1}$, both $\mathfrak{g}$-valued.

Match on-shell d.o.f. in $\mathsf{d} = 3, 4, 6, 10$ if λ is M, M/W, SMW, MW.

Match off-shell d.o.f. using $\mathsf{d} - 3$ bosonic $\mathfrak{g}$-valued auxiliary scalars Y.

Schematically, off-shell SUSY variations of the form

$$\delta_\epsilon A_\mu = \overline{\epsilon}\,\Gamma_\mu \lambda \ , \qquad \delta_\epsilon \lambda = -\tfrac{1}{2} F^{\mu\nu} \Gamma_{\mu\nu} \epsilon + Y \epsilon \ , \qquad \delta_\epsilon Y = \overline{\epsilon}\, \not{D} \lambda$$

where ϵ of same spinorial type as λ.

SUSY variations are scale-invariant; (A_μ, λ, Y) have dimensions $(1, \frac{3}{2}, 2)$.

SUSY algebra closes with $\delta_\epsilon^2 = \xi^\mu \partial_\mu + \delta_\Lambda$, gauge parameter $\Lambda = -\xi^\mu A_\mu$.
(Only w.r.t. $9/16$ supercharges in $\mathsf{d} = 10$ [Berkovits(1993)].)

SUSY lagrangian of the form

$$-\tfrac{1}{4}(F_{\mu\nu}, F^{\mu\nu}) - \tfrac{1}{2}(\overline{\lambda}, \not{D}\lambda) + \tfrac{1}{2}(Y, Y)$$

Now attempt to conformally couple on $\mathscr{M}$:

- Rigid SUSY variations become

$$\delta_\epsilon A_\mu = \bar{\epsilon}\,\Gamma_\mu \lambda \ , \qquad \delta_\epsilon \lambda = -\tfrac{1}{2} F^{\mu\nu} \Gamma_{\mu\nu} \epsilon + Y \epsilon \ , \qquad \delta_\epsilon Y = \bar{\epsilon}\, \not{D} \lambda + \left(\tfrac{\mathsf{d}-4}{\mathsf{d}}\right) \overline{\not{\nabla} \epsilon}\, \lambda$$

- Squaring them gives

$$\delta_\epsilon^2 = \mathcal{L}_\xi + \delta_\sigma + \delta_\rho + \delta_\Lambda$$

off-shell w.r.t. parameters defined above, provided ϵ is a twistor spinor.

- Conformal coupling of lagrangian requires extra compensator φ in $\mathsf{d} \neq 4$.
- Can fix $\varphi = 1$ using a Weyl transformation and take

$$\nabla_\mu \epsilon = -\tfrac{1}{8} \Gamma_\mu \not{\Theta} \epsilon$$

where $\not{\Theta} = \frac{1}{6} \Theta^{\mu\nu\rho} \Gamma_{\mu\nu\rho}$ for some $\Theta \in \bigwedge^3 (\mathscr{M}, \mathbb{R})$.

- In $\mathsf{d} = 3, 6$ can build lagrangian of the form

$$-\tfrac{1}{4}(F_{\mu\nu}, F^{\mu\nu}) - \tfrac{1}{2}(\overline{\lambda}, \not{D}\lambda) + \tfrac{1}{2}(Y, Y) + \tfrac{1}{2}\Theta^{\mu\nu\rho}(A_\mu, \partial_\nu A_\rho + \tfrac{1}{3}[A_\nu, A_\rho]) + \tfrac{1}{8}(\overline{\lambda}, \not{\Theta}\lambda)$$

with rigid SUSY provided Θ is co-closed.

Backgrounds supporting rigid SUSY

Up to local conformal equivalence, in $d \geq 3$, $\mathscr{M}$ which admit (nowhere vanishing) twistor spinors have been classified [Baum+Leitner(2005)]:

- $\mathbb{R}^{1,1} \times$ riemannian manifold admitting parallel spinors.
- Lorentzian Einstein-Sasaki manifold.
- Lorentzian Einstein-Sasaki $\times$ riemannian manifold admitting Killing spinors.
- Fefferman space.
- Brinkmann wave admitting parallel spinor.

– Support rigid SUSY by conformally coupling a SCFT in Minkowski space.

Also gauge theories with rigid SUSY on

ϵ obeys	d	$\mathscr{M}$	Θ	Comment
$\nabla_\mu \epsilon = -\frac{1}{8} \Gamma_\mu \not{\Theta} \epsilon$	3	AdS_3	$\mathrm{vol}_{\mathrm{AdS}_3}$	
	6	$\mathrm{AdS}_3 \times S^3$	$\mathrm{vol}_{\mathrm{AdS}_3} - \mathrm{vol}_{S^3}$	'Freund-Rubin'
$\nabla_\mu \epsilon = \frac{1}{8} \not{\Theta}\, \Gamma_\mu \epsilon$	6	Minimal Poincaré	$H = dB$	[Rigid limit of] Nishino-Sezgin
$(\not{G} \epsilon = \frac{1}{2} \not{\Theta} \epsilon)$	10	SUGRA backgrounds	$(G = d\Phi)$	Chapline-Manton

Closing remarks and outlook

Classify backgrounds supporting maximal rigid SUSY.

– Conformally flat $\mathscr{M}$ + flat projective connection from gauging R-symmetry.

SUSY preserving boundary conditions and dualities.

– Careful treatment only for simple non-compact spaces like AdS.

– $\mathscr{N} = 4$ SYM on $\mathbb{R}^4_+ \rightsquigarrow$ half-BPS SCFTs in 3d [Gaiotto+Witten(2008)].

Geometry of SUSY defects.

– Generalisation of Wilson and 't Hooft operators in higher dimensions.

– Exact results for correlation functions via localisation?

– Reduction to matrix models?

Quantum consistency in curved space.

– $\mathscr{N} = 2$ SCFTs on lorentzian four-manifolds admitting twistor spinors.

– Interesting new backgrounds supporting extended rigid SUSY (e.g. dS_4).

– Tentative finiteness results via algebraic renormalisation.

Happy Birthday, José!

A Timeline in Quadrature
(Well, more or less...)

@	4^2	$\rightsquigarrow$	MIT	[1979]
@	5^2	$\rightsquigarrow$	KU Leuven	[1988]
@	6^2	$\rightsquigarrow$	EMPG	[1999]
@	7^2	$\rightsquigarrow$	???	[2012]

Novel 6d tensor supermultiplet coupling

Minimal on-shell tensor supermultiplet on $\mathbb{R}^{1,5}$ contains

- Two-form gauge field B, with $dB =: H = -*H$ on-shell.
- Fermionic symplectic Majorana-Weyl spinor χ^A, with $\partial\!\!\!/ \chi^A = 0$ on-shell.
- Real bosonic scalar ϕ, with $\Box\phi = 0$ on-shell.

SUSY variations scale-invariant; (B, χ^A, ϕ) have dimensions $(2, \frac{5}{2}, 2)$.

Can couple to off-shell SYM multiplet (with a dimensionless coupling κ)
– conformally coupling the resulting SUSY transformations on $\mathscr{M}$ gives

$$\delta_\epsilon B_{\mu\nu} = \bar{\epsilon}^A \Gamma_{\mu\nu} \chi_A + \kappa\,(A_\mu, \delta_\epsilon A_\nu) - \kappa\,(A_\nu, \delta_\epsilon A_\mu)$$
$$\delta_\epsilon \chi^A = -\tfrac{1}{12}\,\mathscr{H}^{\mu\nu\rho}\Gamma_{\mu\nu\rho}\epsilon^A + \nabla^\mu\phi\,\Gamma_\mu \epsilon^A + \tfrac{\kappa}{2}\,(\delta_\epsilon A_\mu\,, \Gamma^\mu \lambda^A) + \tfrac{2}{3}\phi \nabla\!\!\!\!/\, \epsilon^A$$
$$\delta_\epsilon \phi = \bar{\epsilon}^A \chi_A$$

where $\mathscr{H}_{\mu\nu\rho} = H_{\mu\nu\rho} + 6\kappa\,(A_{[\mu}, \partial_\nu A_{\rho]}) + 2\kappa\,(A_\mu, [A_\nu, A_\rho])$ invariant under gauge variations $\delta_\Lambda B_{\mu\nu} = -2\kappa\,(\Lambda, \partial_{[\mu}, A_{\nu]})$ and $\delta_\Lambda A_\mu = D_\mu \Lambda$.

Rigid SUSY algebra closes up to Weyl-invariant field equations

$\mathscr{H}^{+}_{\mu\nu\rho} = -\frac{\kappa}{4}\,(\bar{\lambda}^{A}, \Gamma_{\mu\nu\rho}\lambda_{A})$, $\qquad \nabla\!\!\!\!/\,\chi^{A} = \kappa\left(\frac{1}{2}(F^{\mu\nu}, \Gamma_{\mu\nu}\lambda^{A}) + (Y^{AB}, \lambda_{B})\right)$

κ-dependent terms in first equation are precisely the Θ-couplings

$$\frac{1}{2}\Theta^{\mu\nu\rho}(A_{\mu}, \partial_{\nu}A_{\rho} + \frac{1}{3}[A_{\nu}, A_{\rho}]) + \frac{1}{8}(\bar{\lambda}, \Theta\!\!\!/\,\lambda)$$

in SYM lagrangian with rigid SUSY parameter obeying $\nabla_{\mu}\epsilon^{A} = -\frac{1}{8}\Gamma_{\mu}\Theta\!\!\!/\,\epsilon^{A}$.

Lagrangian for this conformally coupled gauged tensor supermultiplet is

$$\mathscr{L}_{\rm SYM} + \frac{1}{12\kappa}\,\Theta^{\mu\nu\rho}\left(\mathscr{H}^{+}_{\mu\nu\rho} + \frac{\kappa}{4}\,(\bar{\lambda}^{A}, \Gamma_{\mu\nu\rho}\lambda_{A})\right) - \frac{1}{10\kappa}\,R\phi$$

– Preserves rigid SUSY.

– Correct equations of motion for background tensor supermultiplet.

– Weyl variation of lagrangian gives correct equation of motion for ϕ.