String Theory 2007

Tutorial Sheet 10

AdS/CFT

The following problem emphasizes that arguments leading to the AdS_5/CFT_4 correspondence can also be applied to different D-branes.

Problem 10.1 The mass scale for an open string stretched between any set of parallel Dp-branes is always set by $U = r/\alpha'$. Furthermore, the effective action describing the dynamics of N of these branes is always given by $p+1$ dimensional SuperYang-Mills. It is natural to wonder how the decoupling limit giving rise to the AdS_5/CFT_4 works for other D-branes, since for $p \neq 3$, all these branes have a running dilaton.
Consider the supergravity description for N parallel and coincident D2-branes :

$$\begin{aligned} ds^2 &= f_2^{1/2}\, ds^2(\mathbb{R}^{1,2}) + f_2^{1/2}\, ds^2(\mathbb{R}^7)\,, \\ e^{-2(\phi-\phi_\infty)} &= f_2^{-1/2}\,, \\ C_3 &= -\frac{1}{2}\left(f_2^{-1}-1\right) \mathrm{dvol}\, \mathbb{R}^{1,2}\,, \\ \alpha' f_2 &= \alpha' + d_2 \frac{g_{YM}^2 N}{U^5}\,, \quad g_{YM} = \frac{g_s}{\sqrt{\alpha'}}\,, \quad d_2 = 8\pi^{3/2}\Gamma(5/2)\,. \end{aligned}$$

i. Since the Yang-Mills coupling constant has dimensions, the natural decoupling limit to take is

$$U = \frac{r}{\alpha'} \text{fixed} \quad g_{YM}^2 = \frac{g_s}{\sqrt{\alpha'}} \text{fixed} \quad \alpha' \to 0$$

Show the metric and dilaton for the N D2-branes reduce, under the above decoupling limit, to :

$$ds^2 = \alpha' \left(\frac{U^{5/2}}{g_{YM}\sqrt{6\pi^2 N}}\, ds^2(\mathbb{R}^{1,2}) + \frac{g_{YM}\sqrt{6\pi^2 N}}{U^{5/2}}\, dU^2 + g_{YM}\sqrt{6\pi^2 N/U}\, ds^2(S^6) \right),$$

$$e^{\phi} = \left(\frac{g_{YM}^{10}\, 6\pi^2 N}{U^5} \right)^{1/4}.$$

ii. Given the energy scale U in the gauge theory, the effective *dimensionless* coupling is given by $g_{\text{eff}} \approx g_{YM}^2 N/U$. Conclude that perturbative computations in the gauge theory require to work at energies satisfying

$$U \gg g_{YM}^2 N.$$

Thus, this is a theory which is ultraviolet (UV) free, because going to the UV is equivalent to sending $U \to \infty$.

iii. The supergravity description requires to work at weak coupling, i.e. $e^{\phi} \ll 1$ and at low curvatures, i.e. $\alpha' R \ll 1$. Prove these statements require the Higgs field U to satisfy the inequalities :

$$g_{YM}^2 N^{1/5} \ll U \ll g_{YM}^2 N.$$

It is necessary for $N \gg 1$ for these inequalities to be satisfied, but not sufficient.

iv. Notice the existence of a transition between the perturbative SuperYang-Mills and the supergravity descriptions occurring at $g_{\rm eff} \sim 1$.

v. In the region $U < g_{\rm YM}^2 N^{1/5}$, the dilaton becomes large, but we can to use the eleven dimensional supergravity description whenever its curvature is small in terms of the eleven dimensional Planck scale l_p, i.e. $R\, l_p^2 \ll 1$. Prove this is equivalent to :

$$R\, l_p^2 \sim e^{2\pi/3} \frac{1}{g_{\rm eff}} \sim \frac{1}{N^{1/3}} \left(\frac{g_{\rm YM}^2}{U}\right)^{1/3} \ll 1$$

Thus, for large N, in the region $g_{\rm YM}^2 < U$, the curvature is small in eleven dimensional Planck units.

vi. Assuming that when $U < g_{\rm YM}^2$, the right uplifted solution in eleven dimensions to consider is one in which the N M2-branes are *localised* in the compact direction :

$$f_{M2} = \sum_{n=-\infty}^{\infty} \frac{2^5 \pi^2 N\, l_p^6}{\left(r^2 + (x_{11} - x_{11}^0 + 2\pi\, n\, R_{11})^2\right)^3},$$

with $x_{11} \sim x_{11} + 2\pi R_{11}$, we can see that for very low energies

$$U \ll g_{\rm YM}^2,$$

one is actually probing the spacetime very close to the M2-brane. Thus, in that limit, we can neglect the images in the harmonic function f_{M2}, and the solution resembles that of the near horizon of N M2-branes in a non-compact spacetime, which is conjectured to be dual to a superconformal field theory in 1+2 dimensions with SO(8) symmetry[1]. Notice the transition between a localised and a delocalised M2-brane supergravity solution occurs, roughly, at $U \sim g_{\rm YM}^2$. But at that point, the eleven dimensional radius $R_{11} = g_{\rm YM}^2 \alpha'$ is of order

$$R_{11} \sim l_p N^{1/6} \gg l_p$$

which is much larger than the Planck scale, and so we can still trust the supergravity description.

[1] This statement would follow from the use of the lecture arguments starting from N M2-branes, and giving rise to a near horizon geometry $AdS_4 \times S^7$.