String Theory 2007

Tutorial Sheet 5

Supergravity branes

The following problems deal with the explicit construction of half-BPS bosonic supergravity configurations describing extended objects (branes) in string theory.

Problem 5.1 Let us consider the bosonic lagrangian truncation discussed in our lectures in the particular case of D=11, vanishing dilatonic coupling ($a = 0$) and vanishing dilaton. This corresponds to the a certain truncation of D=11 Supergravity when we choose the degree of the field strength to be four ($n = 4$) :

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-\det g} \left[R - \frac{1}{2 \cdot 4!} |F_4|^2 \right] .$$

Due to the absence of an scalar field, there will be no distinction here between Einstein and string frame.

i. Show that our electric solution gives rise to the M2-brane background

$$\begin{aligned} g &= H^{-2/3} ds^2(\mathbb{R}^{1,2}) + H^{1/3} ds^2(\mathbb{R}^8) \\ F_4 &= dC_3 = \mathrm{dvol}(\mathbb{R}^{1,2}) \wedge dH^{-1} , \\ H &= 1 + \frac{Q_{M2}}{r^6} , \end{aligned}$$

where r stands for the radial coordinate in the transverse space $\mathbb{R}^8$ to the M2-brane located at $r = 0$.

ii. Show that our magnetic solution gives rise to the M5-brane background

$$\begin{aligned} g &= H^{-1/3} ds^2(\mathbb{R}^{1,5}) + H^{2/3} ds^2(\mathbb{R}^5) \\ F_4 &= dC_3 = \star_5 dH , \\ H &= 1 + \frac{Q_{M5}}{r^3} , \end{aligned}$$

where r stands for the radial coordinate in the transverse space $\mathbb{R}^5$ to the M5-brane located at $r = 0$.

Problem 5.2 Let us consider the bosonic lagrangian truncation in D=10 :

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-\det g} \left[R - \frac{1}{2} |\nabla\phi|^2 - \frac{1}{2 \cdot n!} e^{a\phi} |F_n|^2 \right] .$$

By construction, this provides a description of a certain sector of ten dimensional supergravities theories in the Einstein frame, since there is no dilatonic coupling in front of the Hilbert-Einstein term of the action.

i. Consider $n = 3$ and interpret F_3 as the field strength of the NS-NS two form, i.e. $F_3 = H_3 = dB_2$. Comparing the above action with the string frame action :

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-\det g}\,e^{-2\phi}\left[R + 4|\nabla\phi|^2 - \frac{1}{2\cdot 3!}\,|H_3|^2\right],$$

and knowing that $g^e_{mn} = e^{-\phi/2}\,g^s_{mn}$ (where the superscripts e and s stand for Einstein and string frame respectively), fix the value of the constant parameter a in our initial lagrangian. Show that in the string frame, the solution describing a long fundamental string (electric ansatz) is given by :

$$\begin{aligned} g &= H^{-1}\,ds^2(\mathbb{R}^{1,1}) + ds^2(\mathbb{R}^8) \\ B_2 &= H^{-1}\wedge \mathrm{dvol}\,\mathbb{R}^{1,1}\,, \\ e^{-2\phi} &= H\,, \\ H &= 1 + \frac{Q_F}{r^6}\,, \end{aligned}$$

where r stands for the radial coordinate in the transverse space $\mathbb{R}^8$ to the long F-string located at $r = 0$.

ii. In the same theory as above, consider the magnetic ansatz and show the solution in the string frame, so called NS5-brane, is described by :

$$\begin{aligned} g &= ds^2(\mathbb{R}^{1,5}) + H\,ds^2(\mathbb{R}^4) \\ H_3 &= dB_2 = \star_4 dH\,, \\ e^{-2\phi} &= H^{-1}\,, \\ H &= 1 + \frac{Q_{NS5}}{r^2}\,, \end{aligned}$$

where r stands for the radial coordinate in the transverse space $\mathbb{R}^4$ to the NS5-brane located at $r = 0$.

iii. Follow the same steps as above to show that classical Dp-branes supergravity backgrounds in the string frame are given by :

$$\begin{aligned} g &= H^{-1/2}\,ds^2(\mathbb{R}^{1,p}) + H^{1/2}\,ds^2(\mathbb{R}^{9-p}) \\ F^e_{p+2} &= \mathrm{dvol}\,\mathbb{R}^{1,p}\wedge dH^{-1} \quad p = 0,1,2\,, \\ F^m_{8-p} &= \star_{9-p}dH \quad p = 4,5,6\,, \\ e^{-2\phi} &= H^{(p-3)/2}\,, \\ H &= 1 + \frac{Q_{Dp}}{r^{7-p}}\,, \end{aligned}$$

where r stands for the radial coordinate in the transverse space $\mathbb{R}^{9-p}$ to the Dp-brane located at $r = 0$ (Note : the case $p = 3$ was covered in exercise 4.5 in the previous tutorial sheet).