String Theory 2007

Tutorial Sheet 7

Worldvolume solitons

The following problems use the existing non-trivial background couplings in the D-brane worldvolume effective actions to argue the existence of non-trivial bound states in string theory.

Problem 7.1 The Wess-Zumino piece of the D-brane effective action contains a term of the form :

$$\int_{D(p+2)} C_{p+1} \wedge \mathscr{F}\,.$$

Thus, magnetic flux on a D(p+2)-brane can induce Dp-brane charge, because it acts as a source for the corresponding C_{p+1} gauge field to which the Dp-brane couples minimally.
Let us consider a D0-D2 system, as a particular case of the general Dp-D(p+2) bound state. To describe this, take the 1+2 effective theory describing a single D2-brane in ten dimensional Minkowski spacetime and turn on some constant magnetic flux $F = F_{12}$. If the D2-brane is extended along the 12 directions in the bulk, show :

i. $-\det(\mathscr{G} + \mathscr{F}) = 1 + F^2$, where $\mathscr{G}$ stands for the induced worldvolume metric.

ii. the kappa symmetry projector reduces to

$$\sqrt{1+F^2}\,\varepsilon = (\Gamma_{012} + \Gamma_0\,\Gamma_{11}\,F)\,\varepsilon\,,$$

where ε is a constant 32-component Majorana Killing spinor of the ten dimensional Minkoswki spacetime.

iii. the above equation can be solved by

$$F = \tan\alpha\,,$$
$$\varepsilon = (\cos\alpha\,\Gamma_{012} + \sin\alpha\,\Gamma_0\Gamma_{11})\,\varepsilon\,.$$

Notice the projector involves the linear combination of two anticommuting products of gamma matrices, each of which would be describing the individual D-branes that formed the bound state.

Problem 7.2 In the main lectures, we described a long fundamental string ending on a D-brane. The string was transverse to the brane, and this forced us to excite a transverse scalar field to describe such an excitation. Can we have fundamental strings in the D-brane directions ? If so, identify the bosonic configuration that should describe them, and its supersymmetry projection condition. If not, argue why.

Problem 7.3 The Wess-Zumino piece of the D-brane effective action contains a term of the form :

$$\int_{D(p+4)} C_{p+1} \wedge \mathscr{F} \wedge \mathscr{F}\,.$$

Thus, magnetic flux on a D(p+4)-brane can induce Dp-brane charge, because it acts as a source for the corresponding C_{p+1} gauge field to which the Dp-brane couples minimally.

Consider a D0-D4 system, as a particular case of the more general marginal Dp-D(p+4) bound state. To describe this, take the 1+4 effective theory describing the D4-brane in ten dimensional Minkowski spacetime and turn on some magnetic flux $F = \frac{1}{2} F_{ab}\, d\sigma^a \wedge d\sigma^b \; a,b = 1,2,3,4$, keeping all transverse scalar fields to a constant value (not excited). Prove that the supersymmetry of the configuration requires the magnetic field to be self-dual (or antiself-dual).

Problem 7.4 Just as a long fundamental string can end on a Dp-brane breaking 1/4 of the spacetime supersymmetry, we can wonder whether there are other "intersections" of branes that are consistent with supersymmetry.

Consider a D2-brane extended in the 12 spacetime directions. We have learnt how to describe this configuration as the vacuum of a 1+2 effective field theory by turning off the electromagnetic fields on the brane and setting the transverse scalars fields to a constant (the location of the D2-brane in the transverse space). Let us analyse the possibility that a second D2-brane in the 34 directions can exist at the same time, and whether such "excited" configuration preserves any supersymmetry.

From the perspective of the initial 1+2 field theory, we have to turn on two scalar fields $y = X^3(\sigma^1, \sigma^2)$, $z = X^4(\sigma^1, \sigma^2)$, if we want to describe any geometrical excitation in those spacetime directions. Since we are interested in a static configuration (one that does not change in time), we have already assumed that the excitations will be time independent.

i. If we define $\vec{\nabla} = (\partial_1, \partial_2)$, so that its Hodge dual in two dimensions is $\star\vec{\nabla} = (\partial_2, -\partial_1)$, prove that

$$-\det\mathcal{G} = 1 + |\vec{\nabla} y|^2 + |\vec{\nabla} z|^2 + (\vec{\nabla} y \times \vec{\nabla} z)^2 .$$

ii. Prove the supersymmetry projection condition is equivalent to :

$$\sqrt{-\det\mathcal{G}}\,\varepsilon = \left(\Gamma_{012} + \epsilon^{ab}\partial_a y\,\partial_b z\,\Gamma_{034} - \epsilon^{ab}\partial_a X^r\,\Gamma_{0br}\right)\varepsilon .$$

iii. Since we want to interpret the configuration as an intersection of two transverse D2-brane, it is natural to expect the supersymmetry of the configuration to be determined by the projection conditions :

$$\Gamma_{012}\varepsilon = \varepsilon , \qquad \Gamma_{034}\varepsilon = \varepsilon .$$

Show that under these assumptions, the kappa symmetry projection condition is satisfied if

$$\vec{\nabla} y = \star\vec{\nabla} z .$$

iv. The above conditions are equivalent to requiring holomorphicity of $U = y + i\,z$ in terms of the complex coordinates $\sigma^1 \pm i\,\sigma^2$, i.e. they are the Cauchy-Riemann equations for $U(\sigma^1 \pm i\,\sigma^2)$. This is just a very elementary example of the interplay between supersymmetry and holomorphicity.