String Theory 2007

Tutorial Sheet 8

T-duality and toroidal compactification

The following problems deal with toroidal compactifications of string theory and the corresponding T-duality groups.

Problem 8.1 This problems concerns Kałuża–Klein reduction on a circle.

a. Show that the most general lorentzian metric G admitting a one-parameter spacelike group of isometries is given by

$$ds^2 = G_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + e^{2\varphi}(dz + A_\mu dx^\mu)^2 ,$$

where (x^μ, z) are local coordinates adapted to the isometry; that is, the vector field generating the isometry is $Z = \frac{\partial}{\partial z}$ and where $g_{\mu\nu}$, A_μ and φ are independent of z.

b. Compute the Ricci and scalar curvatures of G and write your answer in terms of the Ricci and scalar curvatures of g, the field-strength $F = dA$ and φ.

c. Assuming the orbits of Z are circles parametrised by $z \in [0, 2\pi R]$, write the Einstein–Hilbert action for G in terms of an action over the quotient space. This lower-dimensional action describes a "dilatonic" extension of Einstein–Maxwell theory.

d. Assume that φ is constant. What extra condition must a solution (g, A) of Einstein–Maxwell theory satisfy for it to lift to a Ricci-flat metric G. (*Hint*: What about the φ field equation?)

Problem 8.2 Determine the GSO-projected massless spectrum for the closed NSR string on $\mathbb{R}^{1,8} \times S^1$ at the self-dual radius.

Problem 8.3 Show that the following pairs $(E \to S^2, H)$ of circle bundles over S^2 are topologically dual for some choice of connection (which you must determine):

a. the Hopf fibration $S^3 \to S^2$ with zero H and the trivial bundle $S^2 \times S^1 \to S^2$ with a nonzero H which you have to determine; and

b. the Hopf fibration $S^3 \to S^2$ with H be the bi-invariant 3-form on the Lie group $SU(2) \cong S^3$, and itself.

Problem 8.4 Fill in the details in the derivation of the Buscher rules for how g and B transform under T-duality.