String Theory 2007

Tutorial Sheet 9

M-theory

The following problems emphasize the role of duality transformations as supergravity solution generating techniques and the power of the orbits of the U-duality group to identify different bounds states of branes depending on the U-dual frame under study.

Problem 9.1 Prove that the purely gravitational and half-BPS M-theory backgrounds are consistent with the web of branes derived from Kaluza-Klein, T-duality and S-duality discussed in the lectures (U-duality orbit). To do this, check that these backgrounds are described by a vanishing $F_4 = dC_3 = 0$ and an eleven dimensional metric :

$$g = -dt^2 + d\rho^2 + (H-1)\,(dt + d\rho)^2 + ds^2(\mathbb{R}^9)\,, \quad H = 1 + \frac{P}{r^7} \quad \text{M-wave}$$

$$g = ds^2(\mathbb{R}^{1,6}) + ds^2_{TN}(y) \qquad \text{KK6-brane}$$

$$ds^2_{TN} = H\,d\vec{y}^2 + H^{-1}\left(d\psi + V\right)^2 \quad \text{where} \quad \nabla \times V = \nabla \cdot H\,, \quad H = 1 + \frac{Q_{KK}}{|\vec{y}|}\,,$$

where r stands for the radial coordinate in the transverse $\mathbb{R}^9$ to an M-wave propagating along the ρ direction and $\vec{y}$ stands for a point in $\mathbb{R}^3$ in the four-dimensional Taub-Nut metric that includes a *compact* direction ψ.

Problem 9.2 We have learnt along this set of lectures that there are less supersymmetric configurations of branes, such as $(Dp \perp F)$, $Dp - D(p+2)$, $Dp - D(p+4)$ systems or $(D2 \perp D2(0))$. Use the full set of duality transformations to infere the existence of similar bound states involving M-branes, KK-modes, KK-branes or NS5-branes.

Problem 9.3 All previous configurations saturate a BPS bound that can be derived from the corresponding supersymmetry algebra. Algebraically, these configurations are characterised by having turned on a couple of charges, such that the Clifford valued matrix associated with them either commute or anticommute. Clearly, using induction, this construction can be generalised to an increasing set of non-trivial charges. Can you find examples of algebraic BPS bounds involving three, four or even more brane charges ? Can you provide a physical interpretation for your algebraic result ? [You can do this in the M-algebra, which is algebraically cleaner, since by U-duality, any BPS state found in this way can be mapped into any other analogous BPS state in type II.]