Quantized Cosmological Constant in (1+1)-dimensional Dilaton-Maxwell gravity

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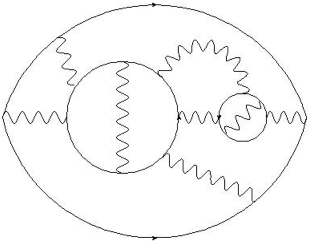
The cosmological constant problem

Strict experimental bound: $|\lambda| \sim 10^{-123}$ in Planck units

Can we predict the value of λ?

Vacuum energy density in QFT $\leftrightarrow$ cosmological constant in GR



$$\rho_{vac} \sim \lambda_{\text{cutoff}}^4$$
$$\Downarrow$$
$$\lambda_{EW} \sim 100\, GeV$$
$$\Downarrow$$
$$\rho_{vac}/\lambda \sim 10^{54}$$

What about gravity?

Gravity in lower dimensions

1d gravity with matter

- Quantization condition on λ
- 1 to 1 correspondence between values for λ and quantum states
- λ takes values in the energy spectrum of the matter sector

(1+1)-dimensional Dilaton Gravity

- Simple (non trivial) model of quantum gravity
- Limit of spherically symmetric GR
- Effective limit of superstring models

The classical model

(1+1)-dimensional Dilaton Gravity with a non-minimal Maxwell field:

$$S_{DM} = \frac{1}{\kappa} \int_{\mathcal{M}} dx^2 \sqrt{-g} \left(XR - U(X) X_{,\mu} X^{,\mu} - 2V(X) - \frac{1}{4} G(X) F_{\mu\nu} F^{\mu\nu} \right)$$

$\mathcal{M}$ compact with no boundaries

Parametrization for the metric tensor:

$$dx^2 = e^{\varphi} \left(-\lambda_0 \lambda_1 dt^2 + (\lambda_0 - \lambda_1) dt\ ds + ds^2 \right)$$

Dilaton dependent conformal transformation:

$$g_{\mu\nu} \to e^{\chi(X)} g_{\mu\nu}$$

Equivalence with Liouville Field Theory

Isolating the cosmological constant: $V(X) = \lambda + \lambda v(X)$

Decoupling conditions $$\begin{cases} U(X) &= \frac{1}{2\xi^2} - \partial_X \ln(v(X)) \\ G(X) &= \frac{2\xi^2 \Lambda_G e^{X/\xi^2}}{\lambda(1+v(X))} \end{cases}$$

New fields $$\begin{cases} \bar{Z} &= \xi\,[\varphi + \chi(X) + \ln(1+v(X))] + \xi\ln(\lambda) \\ \bar{Y} &= \xi\left[\varphi + \chi(X) + \ln(1+v(X)) - X/\xi^2\right] + \xi\ln(\lambda) \end{cases}$$

Extracting the CC from the fields

$$\bar{Z} = Z + \xi\ln(\lambda) \qquad \bar{Y} = Y + \xi\ln(\lambda)$$

The effective action with the cosmological constant

Effective action

$$S_{eff} = \int_{\mathcal{M}} d^2x \frac{\sqrt{-g_\flat}}{\kappa} \left[\frac{1}{2} \left(Z_{,\mu} Z^{,\mu} - Y_{,\mu} Y^{,\mu} \right) - 2\lambda e^{Z/\xi} - e^{-Y/\xi} \frac{\Lambda_G}{2\lambda} F_{\mu\nu} F^{\mu\nu} + \right.$$
$$\left. + \xi \left(Z - Y \right) R_\flat \right]$$

with $\left(g_\flat \right)_{\mu\nu} = g_{\mu\nu} \big|_{\varphi=0}$

This is a gauge independent equivalence!

Covers Liouville Gravity, the Witten Black Hole, the Callan-Giddings-Harvey-Strominger model and more

Equations of motion

Conformal gauge	Coulomb gauge
$\lambda_0 = \lambda_1 = 1$	$A^{\mu}_{,\mu} = 0 \to A_0 = A_{1,s} = 0$

* Liouville fields

$$Y_{,tt} - Y_{,ss} + \frac{\Lambda_G (A_{1,t})^2}{\lambda\xi} e^{-\frac{Y}{\xi}} = 0$$
$$Z_{,tt} - Z_{,ss} - \frac{2e^{\frac{Z}{\xi}}\lambda}{\xi} = 0$$

* Gauge field current equations

$$\partial_\mu \left(A_{1,t} e^{-Y/\xi} \right) = 0 \to E = A_{1,t} e^{-Y/\xi} = const$$

* Two constraints

$$(Y_{,t} \pm Y_{,s})^2 \mp 4\xi \left(Y_{,t} \pm Y_{,s}\right)_{,s} + \frac{2\Lambda_G \left(A_{1,t}\right)^2}{\lambda} e^{-\frac{Y}{\xi}} -$$
$$- \left(Z_{,t} \pm Z_{,s}\right)^2 \pm 4\xi \left(Z_{,t} \pm Z_{,s}\right)_{,s} + 4e^{\frac{Z}{\xi}}\lambda = 0 \ .$$

Additional terms

* The Gibbons-Hawking-York boundary term has a LFT equivalent

$$\int_{\partial\mathcal{M}} dx\,\sqrt{\gamma}XK \quad \Longleftrightarrow \quad \int_{\partial\mathcal{M}} ds\; 2(Z-Y)\left(\xi K_\flat + \xi^2\left(1+8g_\flat\right)Z_{,t}\right)$$

* Minimally coupled *massless* scalar fields are trivially included

$$-\frac{1}{2}\sqrt{-g}\phi_\mu\phi^\mu \quad \Longleftrightarrow \quad -\frac{1}{2}\sqrt{-g_\flat}\phi_{,\mu}\phi^{,\mu}$$

Hamiltonian formulation

Hamiltonian density & constraints

$$\mathcal{H} = \lambda_0 L^+ + \lambda_1 L^- + A_0 L^\emptyset$$

$$L^\pm = -\frac{1}{4}\left(P_Z \mp Z_{,s}\right)^2 \mp \xi\left(P_Z \mp Z_{,s}\right)_{,s} + \lambda e^{Z/\xi} + \\ + \frac{1}{4}\left(P_Y \pm Y_{,s}\right)^2 \mp \xi\left(P_Y \pm Y_{,s}\right)_{,s} + \frac{\lambda}{8\Lambda_G} e^{Y/\xi}\Pi_1^2 + \sum_{i=1}^{D}\frac{1}{4}\left(P_i \pm \phi_{i,s}\right)^2$$

$$L^\emptyset = \Pi_{1,s}$$

Classical algebra

$$\{L^\pm(f), L^\pm(g)\} = \pm L^\pm(fg' - f'g) \approx 0$$

$$\{L^+(f), L^-(g)\} = -\frac{1}{4\Lambda_G}\left(e^{Y/\xi}\Pi_1 L^\emptyset\right)(fg) \approx 0$$

BRST formalism & gauge fixing

- one pair of BRST ghosts per constraint
- BRST charge $\{Q, Q\} = 0$
- BRST extensions: $\mathcal{H}_{brst} = -\{\psi, Q\}$
- gauge fixing $\lambda_0 = \lambda_1 = 1, \; A_0 = \alpha$

gauge fixed BRST extended Hamiltonian (density)

$$\mathcal{H}^{BRST} = L^{+,BRST} + L^{-,BRST} + \alpha L^{\emptyset,BRST}$$

Compactified space dimension $s \in [0, 2\pi)$

e.o.m. admit chiral decomposition for the $L^{\pm}$

Virasoro algebra

$$\{L_n^{\pm,BRST}, L_m^{\pm,BRST}\} = -i(n-m)L_{n+m}^{\pm,BRST}$$

Quantization generalities

General á la Dirac quantization prescriptions:

- Polarization of phase space
- Hilbert space
- Functions $\rightarrow$ (normal ordered) Operators
- Poisson brackets $\rightarrow$ Commutators $\rightarrow$ central charges?
- 1st class (classical) constraints $\rightarrow$ Conditions on physical states

Keeping in mind decoupling:

$$L^{\pm,BRST} = L^{\pm,Z} + L^{\pm,Y} + \sum_{i=1}^{D} L^{\pm,i} + L^{\pm,g}$$

we can quantize each sector separately

Quantum realization of 1st class constraints

Classical Theory		Quantum Theory
1st class constraints	$\Longrightarrow$	conditions on physical states
$L = 0$		$L\|\psi_{phys}\rangle = 0$

Determining λ for physical states

If $|\psi\rangle$ is physical, what is the value of λ?

$$\Downarrow$$

$$\langle\psi_{phys}|L(\lambda)|\psi_{phys}\rangle = 0$$

Equations to be solved for λ and the parameters in ψ_{phys}

The quantum constraints

$$\langle L^+ + L^- \rangle = -\frac{1}{2}\langle P_Z^2 + Z_{,s}^2 \rangle + \frac{1}{2}\langle P_Y^2 + Y_{,s}^2 \rangle + \sum \frac{1}{2}\langle P_i^2 + \phi_{i,s}^2 \rangle +$$

$$+2\xi\langle Z_{,ss} \rangle - 2\xi\langle Y_{,ss} \rangle +$$

$$+\lambda \left[2\langle e^{Z/\xi} \rangle + \frac{1}{4\Lambda_G}\langle e^{Y/\xi}\Pi_1^2 \rangle \right] + O(\hbar)$$

$$\langle L^+ - L^- \rangle = \langle P_Z Z_{,s} \rangle - 2\xi\langle P_{Z,s} \rangle + \langle P_Y Y_{,s} \rangle - 2\xi\langle P_{Y,s} \rangle +$$

$$+\sum\langle P_i \phi_{i,s} \rangle + O(\hbar)$$

$$\langle L^{\emptyset} \rangle = \langle \Pi_{1,s} \rangle + O(\hbar)$$

Compensating contributions from the Z and the Y sectors

Ghost sector quantization

Chiral decomposition

$$c^{\pm}(s) = \sum_{n\in\mathbb{Z}} c_n^{\pm} e^{\mp ins} \qquad b_{\pm}(s) = \sum_{n\in\mathbb{Z}} b_n^{\pm} e^{\mp ins}$$

with $[c_n^{\pm}, b_m^{\pm}]^{+} = \delta_{n+m}$

Radial quantization

$$\left[\hat{L}_n^{+,g}, \hat{L}_m^{+,g}\right] = (n-m)\hat{L}_{m+n}^{+,g} - \frac{13}{6}(n^3-n)\delta_{m+n}$$

$$\left[\hat{L}_n^{-,g}, \hat{L}_m^{-,g}\right] = (n-m)\hat{L}_{m+n}^{-,g} - \frac{13}{6}(n^3-n)\delta_{m+n}$$

$$\left[\hat{L}_n^{+,g}, \hat{L}_m^{-g}\right] = 0$$

Liouville sector quantization

Quantum corrections to the coupling constants

$$\xi \to \xi_Z = \xi + \delta_Z \qquad \xi \to \xi_Y = \xi + \delta_Y$$

$$L^{\pm} = \left[-\frac{1}{4}\left(P_Z \mp Z_{,s}\right)^2 \mp \xi_Z \left(P_Z \mp Z_{,s}\right)_{,s} + \lambda e^{Z/\xi} \right] + \\ + \left[\frac{1}{4}\left(P_Y \pm Y_{,s}\right)^2 \mp \xi_Y \left(P_Y \pm Y_{,s}\right)_{,s} + \frac{\lambda}{8\Lambda_G} e^{Y/\xi} \Pi_1^2 \right] ,$$

Mode expansions

$$Z(s) = \tfrac{i}{2\sqrt{\pi}} \left[a_0 - a_0^\dagger + \textstyle\sum_n{}' \tfrac{1}{n} \left(a_n e^{-ins} + \bar{a}_n e^{ins} \right) e^{-\varepsilon |n|} \right] ,$$

$$P_Z(s) = \tfrac{1}{2\sqrt{\pi}} \left[a_0 + a_0^\dagger + \textstyle\sum_n{}' \left(a_n e^{-ins} + \bar{a}_n e^{ins} \right) e^{-\varepsilon |n|} \right] ,$$

Commutation relations
$[a_n, a_m] = [\bar{a}_n, \bar{a}_m] = n\hbar\, \delta^n_{\ -m},\ a_n^\dagger = a_{-n},\ \bar{a}_n^\dagger = \bar{a}_{-n},\ [a_0, a_0^\dagger] = \hbar.$

Quantum Virasoro algebra in the Liouville sectors

$$\left[L_r^{\pm,Z}, L_q^{\pm,Z}\right] = (r-q)\hbar L_{r+q}^{\pm,Z} + \hbar\left[\left(\frac{\hbar}{24} - 4\pi\left(\xi - \frac{\hbar}{8\pi\xi}\right)^2\right) r^3 + \hbar\frac{1}{12} r\right]\delta_{r+q}$$

$$\left[L_r^{\pm,Y}, L_q^{\pm,Y}\right] = (r-q)\hbar L_{r+q}^{\pm,Y} - \hbar\left[\left(\frac{\hbar}{24} - 4\pi\left(\xi - \frac{\hbar}{8\pi\xi}\right)^2\right) r^3 + \hbar\frac{1}{12} r\right]\delta_{r+q}$$

with the choices

$$\xi_Z = \xi_Y = \xi - \frac{\hbar}{8\pi\xi}$$

.

Additional fields

The Y field with no Maxwell-field

$$L^{\pm,¥} = \left[\frac{1}{4}\left(P_{¥} \pm ¥_{,s}\right)^2 \mp \xi_{¥}\left(P_{¥} \pm ¥_{,s}\right)_{,s}\right]$$

No need to fix $\xi_{¥}$

$$\left[L_r^{\pm,¥}, L_q^{\pm,¥}\right] = (r-q)\hbar L_{r+q}^{\pm,¥} - \hbar\left[\left(\frac{\hbar}{24} - 4\pi\xi_{¥}^2\right) r^3 + \frac{\hbar}{12} r\right]\delta_{r+q}$$

Massless scalar fields

$$\left[L_r^{\pm,\phi}, L_q^{\pm,\phi}\right] = (r-q)\hbar L_{r+q}^{\pm,\phi} - \hbar^2\left(\frac{1}{24} r^3 + \frac{1}{12} r\right)\delta_{r+q}$$

Quantum Virasoro algebra: the case with no Maxwell field

$$L^{\pm,BRST} = L^{\pm,Z} + L^{\pm,\yen} + L^{\pm,g} + \sum_{i=1}^{D} L^{\pm,\phi_i}$$

Central charge

$$c\left(\xi_{\yen}\right) = \hbar \left[\left(-4\pi \left(\xi - \frac{\hbar}{8\pi\xi} \right)^2 + \hbar \frac{52 - D}{24} + 4\pi\xi_{\yen}^2 \right) r^3 + \hbar \frac{26 - D}{12} r \right]$$

Eliminated by fixing $\xi_{\yen}$ (with a possible constraint on D depending on the value of ξ) and shifting the zero modes

$$L_0^{\pm,BRST} \Rightarrow L_0^{\pm,BRST} - \hbar^2 (D - 26)/24$$

Which basis/representation?

- Fock excitations for the states (dropping ghosts contributions)
  allows choices of the states to "test"

$$|\psi^f(d)\rangle = \bigotimes_{n\in\mathbb{Z}} \left[\sum_{\mu\geq 0} d^f_\mu(n) |\mu^f_n\rangle \right]$$

$$|\psi\rangle = \sum_{\{d^Z, d^{¥}, d^i\}} |\psi^Z(d^Z)\rangle |\psi^{¥}(d^{¥})\rangle \bigotimes_{i=1}^{D} |\psi^i(d^i)\rangle$$

- Coherent states basis for operators
  diagonal integral representation of operators

$$\hat{\mathcal{O}} = \int \prod_m \left[\frac{d^2 z_m}{2\pi} \right] |\underline{z}\rangle \mathcal{O}(z, \bar{z}) \langle \underline{z}| \qquad |\underline{z}\rangle = \bigotimes_f^{fields} \left(\bigotimes_n |z^f_n\rangle \right)$$

$$\mathcal{O}(z, \bar{z}) = e^{\left(\sum_m \partial_{z_m} \partial_{\bar{z}_m}\right)} \langle \underline{z}| \hat{\mathcal{O}} |\underline{z}\rangle$$

- Projection on coherent states: $\langle n|z\rangle = \langle\Omega|\mathrm{a}^n|z\rangle = z^n e^{-\frac{1}{2}|z|^2}$
- Subset of states with factorized wave functions: $|\psi|^2 = |\psi^Z|^2|\psi^Y|^2|\psi^1|^2\ldots$
- The constraints are a sum of gaussian integrals over complex variables:

$$\langle L^\pm(s)\rangle = \int\prod_m\left[\frac{dz_m d\bar{z}_m}{2\pi}\right]\text{Ł}^\pm_Z(s,z,\bar{z})|\psi^Z(\underline{z})|^2 + \\ + \int\prod_m\left[\frac{dz_m d\bar{z}_m}{2\pi}\right]\text{Ł}^\pm_{\text{¥}}(s,z,\bar{z})|\psi^{\text{¥}}(\underline{z})|^2 + \\ + \sum_i^D\int\prod_m\left[\frac{dz_m d\bar{z}_m}{2\pi}\right]\text{Ł}^\pm_i(s,z,\bar{z})|\psi^i(\underline{z})|^2$$

The vacuum

The classical cosmological constant needs to vanish

At the quantum level

$$\langle L^+ + L^- \rangle = -\lambda + \frac{\hbar^2}{24}(26 - D) \qquad \langle L^+ - L^- \rangle = 0$$

$$\Downarrow$$

$$\lambda = \frac{26 - D}{24}\hbar^2$$

Quantum correction to
the classical cosmological constant

given by the (reabsorbed) central charge of the quantum algebra
it 'knows' the matter content of the theory

First level excitations

Quantum states

$$|1\rangle = \bigotimes_{f}^{fields} |1^f\rangle = \bigotimes_{f}^{fields} \left[\bigotimes_{n \neq n^f} |\Omega\rangle \right] \otimes \left[d_0^f(n^f)|\Omega\rangle + d_1^f(n^f)|1^f_{n^f}\rangle \right]$$

Non-zero modes constraints

$$\langle L_n^+ + L_{-n}^- \rangle \propto 2\lambda \left(\bar{d}_0^Z(n) d_1^Z(n) + d_0^Z(-n) \bar{d}_1^Z(-n) \right),$$

$$\langle L_n^+ - L_{-n}^- \rangle = \xi_{\yen} \left(\bar{d}_0^{\yen}(n) d_1^{\yen}(n) - d_0^{\yen}(-n) \bar{d}_1^{\yen}(-n) \right) - \\ - \xi_Z \left(\bar{d}_0^Z(n) d_1^Z(n) - d_0^Z(-n) \bar{d}_1^Z(-n) \right),$$

Only pure excitations for Z and $\yen$

$$d_0^Z(n^Z) = d_0^{\yen}(n^{\yen}) = 0$$

Zero modes constraints

$$\langle L_0^+ + L_0^- \rangle = \hbar^2 \frac{26-D}{12} - 2\lambda \left(1 + \frac{|d_1^Z(n^Z)|^2}{4\pi\xi^2|n^Z|}\bigg|_{n^Z\neq 0} \right) + \frac{1}{4\pi}\sum_f^{\text{fields}} \left[\beta(f) \left(2|n^f| + \delta_0^{n^f} \right) |d_1^f(n^f)|^2 \right]$$

$$\langle L_0^+ - L_0^- \rangle \propto \sum_f^{\text{fields}} \left[\beta(f)\; n^f |d_1^f(n^f)|^2 \right]$$

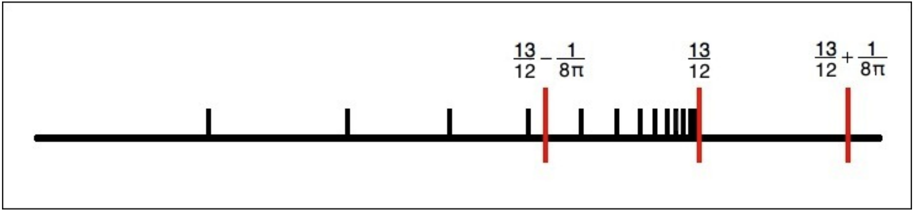
with $\beta = 1$ for matter fields and $\mathrm{\yen}$, and $\beta = -1$ for Z

D=0 first level excitations

1. Both fields are excited, $n^Z = n^{\yen} = N$

$$\lambda = \hbar^2 \frac{13}{12}\left(1 + \frac{1}{4\pi\xi^2 |n^Z|}\bigg|_{n^Z \neq 0}\right)^{-1} = \begin{cases} \hbar^2 \frac{13}{6} \frac{2\pi\xi^2|N|}{1+4\pi\xi^2|N|} & : N \neq 0, \\ \hbar^2 \frac{13}{12} & : N = 0. \end{cases}$$

2. Only the Z field is excited $\quad n^Z = 0 \qquad \lambda = \hbar^2 \frac{13}{12} - \frac{1}{8\pi}$
3. Only the $B^{\emptyset}$ field is excited $\quad n^{\yen} = 0 \qquad \lambda = \hbar^2 \frac{13}{12} + \frac{1}{8\pi}$



Higher excitations

$$D > 0 \text{ gives loosened constraints} \Rightarrow d\text{'s not fixed}$$

* 1st level excitations for $D = 1$
  - the spectrum is bounded from below (the minimum depends on ξ)
  - unbounded towards $+\infty$
  - discrete values + continuous band
* Higher excitations:
  - main difficulty: the $s-$dependence cannot be eliminated with a Fourier transform
  - main interest: contributions from ξ_Z, $\xi_{¥}$ to λ
  - high excitations of Z compensate arbitrary excitations for ¥ and scalar fields

Summary and conclusions

- in 1+1 dimensions a gauge independent duality exists between a class of models of dilaton-Maxwell gravity and Liouville Field Theory
- the cosmological constant is fixed by space-time symmetry in the quantum theory
- the quantum gravitational d.o.f. contributes with a negative term VS positive matter contributions
- the cosmological constant spectrum is 'built' around the central charge of the Virasoro algebra

Thank you!

References arXiv:1102.4957 arXiv:1111.1612