Non-abelian action of M5-Branes

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based on

1. Non-abelian Action for Multiple M5-Branes, with Sheng-Lan Ko, arXiv:1203.4224
2. A Theory of Non-Abelian Tensor Gauge Field with Non-Abelian Gauge Symmetry $G \times G$, arXiv:1108.5131

Outline

Reasons to study M5-branes (flat)

- The low energy worldvolume dynamics is expected to be given by a 6d (2,0) SCFT with $SO(5)$ R-symmetry.

  (Strominger, Witten)

  The (2,0) tensor multiples contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions.

  (Gibbons, Townsend; Strominger; Kaplan, Michelson)

However, other than the global symmetries and the spectrum, it is mysterious!

- Gauge symmetry for multiple M5-branes ?

  (c.f. Yang-Mills gauge symmetry for D-branes)
- Interacting self-dual dynamics on M5-branes worldvolume?

Also,

- Quantized geometry for M5-brane in a large constant 3-form C-field?

  (c.f. Moyal geometry $[x^i, x^j] = i\theta^{ij}$ for D-branes)

  (Chu, Sembi)
- Entropy counting N^3 for N number of M5-branes?

  (c.f. N^2 counting for D-branes)

  (Klebnov, Tseytlin).

Enhanced gauge symmetry of multiple M5-branes

- When a number of D-branes are put together to be in coincidence with each other, the symmetry is enhanced from $U(1)$ to $U(N)$:

$$\delta A_\mu^a = \partial_\mu \Lambda^a + [A_\mu, \Lambda]^a, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + [A_\mu, A_\nu]^a.$$

- For a single M5-brane, worldvolume $B_{\mu\nu}$ has the tensor gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu$$

and the field strength

$$H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$$

is invariant.

- It is not known how to non-Abelianize 2-form (or higher form) gauge fields:

$$\delta B_{\mu\nu}^a = \partial_\mu \Lambda_\nu^a - \partial_\nu \Lambda_\mu^a + (?), \quad H_{\mu\nu\lambda}^a = \partial_\mu B_{\nu\lambda}^a + \partial_\nu B_{\lambda\mu}^a + \partial_\lambda B_{\mu\nu}^a + (?).$$

- In fact, not even know if it is needed. Classically, only requirement is that we want to have nontrivial self interaction.

Self-dual dynamics for multiple M5-branes

- It is well known to be difficult to write down a Lorentz invariant action for self-dual dynamics.

  (Siegel 84; Floreanini, Jackiw 87)

- For a single M5 case, problem solved by Perry-Schwarz (also Henneaux-Teitelboim) by sacrificing manifest 6d Lorentz symmetry.

  (Perry-Schwarz 97; Henneaux-Teitelboim 88)

  Covariant construction given later by PST

  (Pasti-Tonin-Sorokin)

- No clear how to do this for $N > 1$ due to the other problem that an appropriate generalization of the tensor gauge symmetry was not known.
- Moreover, exists no-go theorems: there is no nontrivial deformation of the Abelian 2-form gauge theory if locality of the action and the transformation laws are assumed.

  (Henneaux; Bekaert; Sevrin; Nepomechie)
- The no-go theorems suggest an important direction of given up locality.

- To construct a (bosonic) theory of M5-branes, need to (at least) solve both problems of gauge symmetry and self-duality!

Recent works in this direction:

- Ho, Huang, Matsuo: self-dual theory on compactified $R^5 \times S^1$.
  (compact)
- Samtleben, Sezgin, Wimmer: (1,0) SCF EOM based on tensor hierarchy.
  (susy)
- Chu: tensor gauge symmetry with $G \times G$ gauge bosons.
  (manifest Lorentz symmetry and tensor gauge symmetry)

Also Douglas; Lambert, Papageorgakis, Schmidt-Summerfield:
5d SYM is the multiple M5-theory.

$1+5$ description of Abelian tensor gauge field in 6d

- The abelian field strength is given by

$$H_{MNP} = \partial_M B_{NP} + \partial_N B_{PM} + \partial_P B_{MN} := \partial_{[M} B_{NP]}$$

- Hodge dual:

$$\tilde{G}_{MNP} := -\frac{1}{6}\epsilon_{MNPQRS}\, G^{QRS}.$$

- The self-duality equation reads

$$\tilde{H}_{MNP} = H_{MNP}.$$

Perry-Schwarz formulation

- In the Perry-Schwarz formulation, x_5 is singled out and the self-dual tensor gauge field is represented by a 5×5 antisymmetric tensor field $B_{\mu\nu}$. i.e. $B_{\mu 5}$ never appear.
- Denote the 5d and 6d coordinates by x^μ and $x^M = (x^\mu, x^5)$.
$\eta^{MN} = (-+++++), \epsilon^{01234} = -\epsilon_{01234} = 1, \quad \epsilon^{012345} = -\epsilon_{012345} = 1$
- The Perry-Schwarz action is

$$S_0(B) = \frac{1}{2}\int d^6x \left(-\tilde{H}^{\mu\nu}\tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu}\partial_5 B_{\mu\nu}\right)$$

where

$$\tilde{H}^{\mu\nu} := \frac{1}{6}\epsilon^{\mu\nu\rho\lambda\sigma}H_{\rho\lambda\sigma}, \qquad H^{\mu\nu\rho} = -\frac{1}{2}\epsilon^{\mu\nu\rho\lambda\sigma}\tilde{H}_{\lambda\sigma}.$$

- The equation of motion

$$\epsilon^{\mu\nu\rho\lambda\sigma}\partial_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

is second order and has the general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \partial_\mu\alpha_\nu - \partial_\nu\alpha_\mu, \quad \text{for arbitrary } \alpha_\mu.$$

- The action is invariant under the gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \quad \text{for arbitrary } \varphi_\mu.$$

This allows one to reduce the general solution to the EOM to the first order form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

This is the self-duality equation in this theory.

Modified Lorentz symmetry

- The action is manifestly 5d Lorentz invariant. What about the Lorentz symmetry mixing the μ directions with the 5 direction?

Lorentz transformation (active view)

Standard Lorentz transformation has a orbital part:

$$\Lambda \cdot L = (\Lambda \cdot x)\partial_5 - x_5(\Lambda \cdot \partial)$$

and a spin part:

$$\delta B_{\mu\nu} = \Lambda_\nu B_{\mu 5} - \Lambda_\mu B_{\nu 5}.$$

- PS proposed the modified Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu},$$

where $\Lambda_\mu = \Lambda_{5\mu}$ denote the corresponding infinitesimal transformation parameters.

1. The action is invariant
2. On shell, it is equal to the standard Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu} = (\Lambda \cdot x)\partial_5 B_{\mu\nu} - x_5(\Lambda \cdot \partial)B_{\mu\nu}$$

3. Commutator:

$$[\delta_{\Lambda_1}, \delta_{\Lambda_2}]B_{\mu\nu} = \delta^{(5d)}_{\Lambda_{\alpha\beta}} B_{\mu\nu} + \text{EOM} + \text{gauge symmetry}$$

where

$$\delta^{(5d)}_{\Lambda_{\alpha\beta}} B_{\mu\nu} = \Lambda_\mu{}^\lambda B_{\lambda\nu} - \Lambda_\nu{}^\lambda B_{\lambda\mu} + x_\lambda \Lambda^{\lambda\alpha}\partial_\alpha B_{\mu\nu}$$
$$\Lambda_{\mu\nu} = \Lambda_{1\mu}\Lambda_{2\nu} - \Lambda_{1\nu}\Lambda_{2\mu},$$

and

$$\delta B_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu, \qquad \varphi_\nu = x^\alpha \Lambda_{\alpha\lambda} B_\nu{}^\lambda$$

is the gauge symmetry of the PS theory.

Remarks on modified Lorentz symmetry:

0. Modified Lorentz symmetry is typical of action of self-dual dynamics (Siegel 84)

1. Boundary condition:

$$\partial_\lambda B_{\mu\nu} \to 0 \text{ as } |x^M| \to \infty \ .$$

   is required in the PS model: in establishing the gauge symmetry (hence the self-duality) and the Lorentz symmetry of the theory.

2. One may combine the modified Lorentz transformation with the gauge transformation and obtain an equivalent form of the modified Lorentz transformation

$$\delta B_{\mu\nu} = (\Lambda \cdot x)\tilde{H}_{\mu\nu} - x_5 \Lambda^\kappa H_{\kappa\mu\nu},$$

   which is written entirely in terms of the field strength.

3. That it is possible to support 6d Lorentz symmetry without introducing the components $B_{\mu 5}$ is entirely due to the existence of the gauge symmetry in the theory.

Non-abelian action

Ideas:

- Give up manifest 6d Lorentz symmetry
- Introduce a set of 1-form gauge fields for a gauge group G as suggested by $G \times G$ construction of tensor gauge symmetry

  (Chu)
- Represent the self-dual tensor gauge field by a 5×5 antisymmetric field $B^a_{\mu\nu}$ in the adjoint.

Will not consider:

- supersymmetry
- PST like covariantization

The action S_0

- Consider non-abelian generalization of the Perry-Schwarz action

$$S_0 = \frac{1}{2} \int d^6 x \, \mathrm{tr} \left(-\tilde{H}^{\mu\nu} \tilde{H}_{\mu\nu} + \tilde{H}^{\mu\nu} \partial_5 B_{\mu\nu} \right),$$

where

$$H_{\mu\nu\lambda} = D_\mu B_{\nu\lambda} + D_\nu B_{\lambda\mu} + D_\lambda B_{\mu\nu}$$

and

$$D_\mu = \partial_\mu + A_\mu.$$

- No $B_{\mu 5}$ and A_5.
- A_μ lives in 5-dimensions

- The action S_0 is invariant under:
  Yang-Mills gauge symmetry

$$\begin{aligned} \delta A_\mu &= \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \text{for arbitrary } \Lambda = \Lambda(x^\lambda), \\ \delta B_{\mu\nu} &= [B_{\mu\nu}, \Lambda], \quad \delta H_{\mu\nu\lambda} = [H_{\mu\nu\lambda}, \Lambda] \end{aligned}$$

  "Tensor gauge symmetry":

$$\begin{aligned} \delta_T A_\mu &= 0, \\ \delta_T B_{\mu\nu} &= \Sigma_{\mu\nu}, \quad \text{for arbitrary } \Sigma_{\mu\nu}(x^M) \text{ such that } D_{[\lambda}\Sigma_{\mu\nu]} = 0. \end{aligned}$$

  under the assumption that covariant derivatives vanish at infinity:

$$D_\lambda B_{\mu\nu}\,,\ \partial_5 B_{\mu\nu} \to 0 \quad \text{as} |x^M| \to \infty.$$

- The tensor gauge symmetry is abelian $[\delta_{T^{(1)}}, \delta_{T^{(2)}}] = 0$! Nevertheless the system will be fully interacting.

The action S_E

- Vanishing of field strength at infinity: $H_{\mu\nu\lambda} \to 0$ suggests to identify $F_{\mu\nu}$ is identified with the boundary value of $B_{\mu\nu}$.
- With the anticipation of the self-duality equation of motion in the theory, we will consider the constraint

$$F_{\mu\nu} = \int dx_5 \; \tilde{H}_{\mu\nu}$$

and implement it with a 5d auxiliary field $E_{\mu\nu}$ and the action

$$S_E = \int d^5x \; \mathrm{tr} \left((F_{\mu\nu} - \int dx_5 \; \tilde{H}_{\mu\nu}) E^{\mu\nu} \right) .$$

- S_E is invariant under the Yang-Mills and tensor gauge transformation

$$\delta E_{\mu\nu} = [E_{\mu\nu}, \Lambda], \quad \delta_T E_{\mu\nu} = 0$$

- The action is also invariant under the gauge symmetry

$$\delta E_{\mu\nu} = \alpha_{\mu\nu}$$

for arbitrary $\alpha(x^\lambda)$ such that $D_{[\mu}\alpha_{\nu\lambda]} = 0, \quad D^\mu \alpha_{\mu\lambda} = 0.$

Properties: self-duality

- EOM of $E_{\mu\nu}$ give

$$F_{\mu\nu} = \int dx_5 \ \tilde{H}_{\mu\nu}.$$

- EOM of $B_{\mu\nu}$ gives

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} + E_{\lambda\sigma}) = 0,$$

  Integrating it over x_5, one get $D_{[\rho} E_{\lambda\sigma]} = 0$ and so

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

- This has general solution

$$\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma} = \Phi_{\lambda\sigma},$$

  where $D_{[\lambda}\Phi_{\mu\nu]} = 0$.
- Therefore with an appropriate fixing of the tensor gauge symmetry, one can always reduce the second order EOM to the self-duality equation

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

- EOM of A_μ gives

$$\begin{aligned} D^\mu E_{\mu\nu} &- \frac{1}{4}\int dx_5\ \epsilon_\nu{}^{\alpha\beta\gamma\delta}[B_{\alpha\beta}, E_{\gamma\delta}] \\ &= -\frac{1}{2}\int dx_5\ \epsilon_\nu{}^{\alpha\beta\gamma\delta}[B_{\alpha\beta}, \partial_5 B_{\gamma\delta} - \frac{1}{2}\tilde{H}_{\gamma\delta}] := J^\nu. \end{aligned}$$

Properties: Degrees of freedom

Counting of dof in the free PS action

- PS theory initially has the EOM

$$\epsilon^{\mu\nu\rho\lambda\sigma} D_\rho(\tilde{H}_{\lambda\sigma} - \partial_5 B_{\lambda\sigma}) = 0$$

Using the gauge symmetry

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu,$$

one can fix the equation of motion to the linear form

$$\tilde{H}_{\mu\nu} = \partial_5 B_{\mu\nu}.$$

This leaves us with a x^5-independent residual symmetry.

- Now $\partial^\mu B_{\mu\nu}$ is x_5 independent onshell and so can be gauge fixed to zero

$$\partial^\mu B_{\mu\nu} = 0.$$

This gives 4 independent conditions on the 10 components of $B_{\mu\nu}$.

- Self-duality then gives 3 degrees of freedom.

- For the nonabelian case, treats the higher order terms as interaction and count the degrees of freedom using the linearized theory.
- At the quadratic level, the non-abelian action is simply given by $\dim G$ copies of the Perry-Schwarz action, plus the action S_E.
- We obtain $3 \times \dim G$ degrees of freedom in $B_{\mu\nu}$.
- Expect no degrees of freedom in E as it is an auxillary field and could be integrated out directly. In fact, the linearized equations of motion are

$$\partial_{[\mu} E_{\nu\lambda]} = 0, \quad \partial^{\mu} E_{\mu\nu} = 0,$$

  We can use the gauge symmetry to remove the $E_{\mu\nu}$ field completely.
- Our theory contains $3 \times \dim G$ degrees of freedom as required by (2,0) supersymmetry

Properties: Lorentz Symmetry

- For a general variation,

$$\delta S_0 \quad = \quad \int d^6x \ \mathrm{tr}\left[\Delta B^{\mu\nu}\tilde{H}_{\mu\nu}\right],$$

  where

$$\Delta B^{\mu\nu} := \partial_5(\delta B^{\mu\nu}) - \frac{1}{2}\epsilon^{\mu\nu\alpha\beta\gamma}D_\alpha(\delta B_{\beta\gamma}).$$

- Taking it to be the 5-μ Lorentz transformation of the form:

$$\delta B_{\mu\nu} = (\Lambda\cdot x)\tilde{H}_{\mu\nu} - \lambda x_5\Lambda^\kappa H_{\kappa\mu\nu} + \Lambda^\kappa\phi_{\mu\nu\kappa} := \delta_{(1)}B_{\mu\nu} + \delta_{(2)}B_{\mu\nu},$$

  where λ is a constant, we find

$$\delta_{(1)}S_0 = \int\left[\frac{\lambda}{2}x_5\epsilon^{\mu\nu\alpha\beta\gamma}D_\alpha H_{\beta\gamma\kappa}\Lambda^\kappa + \frac{\lambda-1}{4}\Lambda_\rho\tilde{H}_{\alpha\beta}\epsilon^{\rho\alpha\beta\mu\nu}\right]\tilde{H}_{\mu\nu}.$$

  Thus S_0 is invariant if

$$\partial_5\phi_{\mu\nu\kappa} - \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta\gamma}D_\alpha\phi_{\beta\gamma\kappa} = -\frac{\lambda}{2}x_5\epsilon^{\mu\nu\alpha\beta\gamma}D_\alpha H_{\beta\gamma\kappa} - \frac{\lambda-1}{4}\tilde{H}^{\alpha\beta}\epsilon_{\kappa\alpha\beta\mu\nu} := J_{\mu\nu\kappa}.$$

$\phi_{\mu\nu\kappa}$ can be solved with a Green function method.

- Let $G^{ab}_{\mu\nu,\mu'\nu'}(x,y)$ be the Green function which satisfies

$$\partial_5 G^{ab}{}_{\mu\nu}{}^{\mu'\nu'} - \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta\gamma}(D^{(y)}_\alpha)^a{}_c G^{cb}{}_{\beta\gamma}{}^{\mu'\nu'} = \delta^{\mu'\nu'}_{\mu\nu}\delta^{ab}\delta^{(6)}(x-y)$$

  and the BC

$$G^{ab}{}_{\mu\nu}{}^{\mu'\nu'}(x,y) = 0, \quad |x_5| \to \infty.$$

  Here $x = (x^M)$ and $(D_\alpha)^a{}_c = \partial_\alpha \delta^a{}_c + (\tilde{A}_\alpha)^a{}_c$ where $(\tilde{A}_\alpha)^{ac} := f^{abc}A^b_\alpha$.
  Then

$$\phi^a_{\mu\nu\kappa} = \int dy\ G^{ab}{}_{\mu\nu}{}^{\mu'\nu'}(x,y) J^b_{\mu'\nu'\kappa}(y)$$

- This works for any λ. However to make S_E invariant, we need to take $\lambda = -1$ and if $E_{\mu\nu}$ transforms as

$$\delta E_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta\gamma} D_\alpha((\Lambda\cdot x)E_{\beta\gamma}).$$

Comments

1a. Lorentz invariance of the action implies that the EOM are automatically Lorentz invariant. But note that this applies to the un-gauge fixed EOM, but not the self-duality equation.

- This is not surprising. For example, Yang-Mills EOM in the Coulomb gauge is not Lorentz invariant.
- The use of the self-duality equation is important for obtaining the correct counting on the degrees of freedom in the theory. However the use of the ungauge-fixed version may be useful for some other purposes, for example, supersymmetry.

2. The action and the Lorentz symmetry we proposed are nonlocal. But is needed for multiple M5-branes.

- We are working in a formulation without $B_{\mu 5}$., it is possible that these nonlocalities are due to the fact that we are working in a gauge fixed version of a covariant formulation (exactly like QED in Coulomb gauge or string in lightcone gauge).
- Would be interesting to covariantize our construction (PST like). It is possible that the employment of additional auxiliary fields would allow for a local representation of the Lorentz symmetry.

Properties: Reduction to D4-branes

- Consider a compactification of x_5 on a circle of radius R. The dimensional reduced action reads

$$S = \frac{2\pi R}{2} \int d^5x \,\mathrm{tr} \left(-\tilde{H}_{\mu\nu}^2 + (F_{\mu\nu} - 2\pi R \tilde{H}_{\mu\nu}) E^{\mu\nu} \right)$$

- Integrate out $E_{\mu\nu}$, we obtain

$$F_{\mu\nu} = 2\pi R \tilde{H}_{\mu\nu}.$$

and eliminate $\tilde{H}_{\mu\nu}$, we obtain the 5d Yang-Mills action

$$S_{YM} = -\frac{1}{4\pi R} \int d^5x \,\mathrm{tr}\, F_{\mu\nu}^2.$$

- The action S_{YM} corresponds to the expected form of the YM coupling

$$g_{YM}^2 = R$$

and the gauge group in our construction is to be

$$G = U(N)$$

for a system of N M5-branes.

- But EOM gives $D^\mu F_{\mu\nu} = 0$ instead of

$$D_\mu F^{\mu\nu} = -\frac{\pi R}{2}\epsilon^{\nu\alpha\beta\gamma\delta}[F_{\alpha\beta}, B_{\gamma\delta}]?$$

- Need to be more careful with the implementation of Delta function:

$$\int [DA][DB][DE]e^{-S} = \int [DA][DB]e^{-S_{YM}}\delta(F_{\mu\nu} - 2\pi R\tilde{H}_{\mu\nu}) = \int [DA]e^{-S_{YM}-S'},$$

where consistency requires that

$$\frac{\delta S'}{\delta A_\nu} = \frac{1}{2}\epsilon^{\nu\alpha\beta\gamma\delta}[F_{\alpha\beta}, B_{\gamma\delta}]$$

- The 5d theory is thus given by the action $S_{5d} = S_{YM} + S'$.
S' describes a high derivative correction term to the Yang-Mills theory since $[F, B] \sim DDB$ and B is of the order of F.
Q: captures the non-abelian DBI action of D4-branes?
(Tseytlin; Koerber, Sevrin)

- We have constructed a non-abelian action of tensor fields with the properties:
  1. the action admits a self-duality equation of motion,
  2. the action has manifest 5d Lorentz symmetry and a modified 6d Lorentz symmetry,
  3. on dimensional reduction, the action gives the 5d Yang-Mills action plus corrections.

  Based on these properties, we propose our action to be the bosonic theory describing the gauge sector of coincident M5-branes in flat space.
- A special feature of our theory is that the tensor gauge symmetry is abelian although the theory is still fully interacting. This is different from the self-interaction of YM.

Further questions

- Covariant PST extension of our model?
- Supersymmetry: (2,0)? (1,0)?
- Scalar potential and BPS equation?
- Any connection with the 5d SYM proposal of Douglas and Lambert, Papageorgakis, Schmidt-Summerfield"?
  Where is the B-field in SYM description? (similar to the problem of extracting the gravity field in the BFSS matrix model?)