Lovelock theory, black holes and holography

José D. Edelstein

Universidade de Santiago de Compostela, Spain

CECs, Valdivia, Chile

Maxwell Institute for Mathematical Sciences

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Xián Camanho — arXiv:0911.3160, 0912.1944, 1103.3669 & to appear

Xián Camanho and Miguel Paulos — arXiv:1010.1682

Xián Camanho, Gastón Giribet and Andrés Gomberoff — arXiv:1204.6737 & to appear

Higher curvature corrections and quantum gravity

Classical gravity seems well-described by the Einstein-Hilbert action.

Quantum corrections generically involve higher curvature corrections:

- Wilsonian approaches.
- α' and/or g_s corrections in string theory.
- Higher dimensional scenarios.
- Relevant when studying generic strongly coupled CFTs under the light of the gauge/gravity correspondence (*e.g.*, 4d CFTs with $a \neq c$).

They are typically argued to be plagued of ghosts.

Lovelock gravities are the most general second order theories free of ghosts when expanding about flat space. Lovelock (1971)

Lovelock theory

The action is compactly expressed in terms of differential forms

$$\mathcal{I} = \sum_{k=0}^{K} \frac{c_k}{d-2k} \left(\int_{\mathcal{M}} \mathcal{I}_k - \int_{\partial\mathcal{M}} \mathcal{Q}_k \right)$$

where $K \leq [\frac{d-1}{2}]$ and c_k is a set of couplings with length dimensions $L^{2(k-1)}$.

- $\mathcal{I}_k$ is the extension of the Euler characteristic in $2k$ dimensions

$$\mathcal{I}_k = \epsilon_{a_1 \cdots a_d} \, R^{a_1 a_2} \wedge \cdots \wedge R^{a_{2k-1} a_{2k}} \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_d}$$

  with $R^{ab} = d\omega^{ab} + \omega^a{}_c \wedge \omega^{cb} = \frac{1}{2} R^{ab}{}_{\mu\nu} \, dx^\mu \wedge dx^\nu$.

- $\mathcal{Q}_k$ comes from the GB theorem in manifolds with boundaries Myers (1987)

$$\mathcal{Q}_k = k \int_0^1 d\xi \, \epsilon_{a_1 \cdots a_d} \, \theta^{a_1 a_2} \wedge \mathfrak{F}^{a_3 a_4}(\xi) \wedge \cdots \wedge \mathfrak{F}^{a_{2k-1} a_{2k}}(\xi) \wedge e^{a_{2k+1}} \wedge \cdots \wedge e^{a_d}$$

  where $\theta^{ab} = n^a K^b - n^b K^a$ and $\mathfrak{F}^{ab}(\xi) \equiv R^{ab} + (\xi^2 - 1)\, \theta^a{}_e \wedge \theta^{eb}$.

Lovelock theory: lowest order examples

The first two contributions (most general up to $d = 4$) correspond to:

- The cosmological term: we set $2\Lambda = -\dfrac{(d-1)(d-2)}{L^2}$ $\qquad c_0 = \dfrac{1}{L^2}$
- The EH action (with GH term): we set $16\pi(d-3)!G_N = 1$ $\qquad c_1 = 1$

For $d \geq 5$, we have the Lanczos-Gauss-Bonnet (LGB) term ($c_2 = \lambda\, L^2$),

$$\mathcal{I}_2 \simeq d^d x\, \sqrt{-g}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) \qquad \mathcal{Q}_2 \sim \sqrt{-h}\,(KR + \ldots)$$

while for $d \geq 7$, the cubic Lovelock Lagrangian ($c_3 = \mu\, L^4$),

$$\begin{aligned} \mathcal{I}_3 \quad \simeq \quad & d^d x\, \sqrt{-g}\Big(R^3 + 3RR^{\mu\nu\alpha\beta}R_{\alpha\beta\mu\nu} - 12RR^{\mu\nu}R_{\mu\nu} + 24R^{\mu\nu\alpha\beta}R_{\alpha\mu}R_{\beta\nu} + \\ & 16R^{\mu\nu}R_{\nu\alpha}R_{\mu}^{\ \alpha} + 24R^{\mu\nu\alpha\beta}R_{\alpha\beta\nu\rho}R_{\mu}^{\ \rho} + 8R^{\mu\nu}_{\ \ \ \alpha\rho}R^{\alpha\beta}_{\ \ \ \nu\sigma}R^{\rho\sigma}_{\ \ \ \mu\beta} + 2R_{\alpha\beta\rho\sigma}R^{\mu\nu\alpha\beta}R^{\rho\sigma}_{\ \ \ \mu\nu}\Big) \end{aligned}$$

AdS/dS/flat vacua

Varying the action with respect to the connection,

$$\epsilon_{aba_3\cdots a_d} \sum_{k=1}^{K} \frac{k\, c_k}{d-2k} \left(R^{a_3 a_4} \wedge \cdots \wedge R^{a_{2k-1} a_{2k}} \wedge e^{a_{2k+1}} \wedge \ldots \wedge e^{a_{d-1}}\right) \wedge T^{a_d} = 0$$

we can safely impose $T^a = 0$ as in the standard Einstein gravity.

The equations of motion, when varying with respect to the vierbein,

$$\epsilon_{aa_1\cdots a_{d-1}}\, \mathcal{F}_{(1)}^{a_1 a_2} \wedge \cdots \wedge \mathcal{F}_{(K)}^{a_{2K-1} a_{2K}} \wedge e^{a_{2K+1}} \wedge \ldots \wedge e^{a_{d-1}} = 0$$

admit K constant curvature *vacua*,

$$\mathcal{F}_{(i)}^{ab} := R^{ab} - \Lambda_i\, e^a \wedge e^b = 0$$

The cosmological constants being the roots of the polynomial $\Upsilon[\Lambda]$:

$$\Upsilon[\Lambda] := \sum_{k=0}^{K} c_k\, \Lambda^k = c_K \prod_{i=1}^{K} (\Lambda - \Lambda_i) = 0$$

Degeneracies arise when $\Delta := \prod_{i<j} (\Lambda_i - \Lambda_j)^2 = 0$

Warming up: the LGB case

When $K = 2$:

$$\Lambda_\pm = -\frac{1 \pm \sqrt{1-4\lambda}}{2\lambda L^2} \qquad \text{then} \qquad \Delta = 0 \quad \Leftrightarrow \quad \lambda = \lambda_{\rm CS} := \frac{1}{4}$$

- For $0 < \lambda < \lambda_{\rm CS}$: two AdS vacua; the $+$ sign is unstable. Boulware, Deser (1985)
- For $\lambda = \lambda_{\rm CS}$ the theory displays symmetry enhancement.
- For $\lambda > \lambda_{\rm CS}$ there is no AdS vacuum.

The EH-branch has $\Upsilon'[\Lambda_-] > 0$, a positive effective Newton constant.

This latter result can be generalized to arbitrary Lovelock gravities,

$$\boxed{\Upsilon'[\Lambda_\star] > 0}$$

being required for gravitons with the right sign in the kinetic term.

Maldacena's conjecture: the AdS/CFT correspondence

Bold statement: Maldacena (1997)

Quantum gravity in AdS space is equal to a CFT living at the boundary

For example, type IIB superstring theory in $\mathrm{AdS}_5 \times S^5$. Notice that

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2 + d\vec{x}^2 + dz^2\right] + L^2\, d\Omega_5^2$$

whose isometry group is $SO(4,2) \times SO(6) \subset PSU(2,2|4)$ of $\mathcal{N} = 4$ SYM.

A dictionary has to be established:

- The radial direction, z, in AdS is the energy scale of the CFT.
- The generating function reads

Gubser, Klebanov, Polyakov (1998)
Witten (1998)

$$\mathcal{Z}_{\mathrm{QG}}\left[g_{\mu\nu}\right] \approx \exp\left(-I_G[g_{\mu\nu}]\right) = \left\langle \exp\left(\int d\mathbf{x}\, \eta^{ab}(\mathbf{x})\, T_{ab}(\mathbf{x})\right)\right\rangle_{\mathrm{SYM}}$$

where $g_{\mu\nu} = g_{\mu\nu}(z, \mathbf{x})$ such that $g_{ab}(0, \mathbf{x}) = \eta_{ab}(\mathbf{x})$.

Consider a CFT_{d-1}. The leading singularity of the 2-point function is fully characterized by the central charge C_T Osborn, Petkou (1994)

$$\langle T_{ab}(\mathbf{x})\, T_{cd}(\mathbf{0})\rangle = \frac{C_T}{2\,\mathbf{x}^{2(d-1)}} \left(I_{ac}(\mathbf{x})\, I_{bd}(\mathbf{x}) + I_{ad}(\mathbf{x})\, I_{bc}(\mathbf{x}) - \frac{1}{d-1}\,\eta_{ab}\,\eta_{cd} \right)$$

$$\text{where} \qquad I_{ab}(\mathbf{x}) = \eta_{ab} - 2\,\frac{x_a\, x_b}{\mathbf{x}^2}$$

The holographic computation of C_T gives Camanho, Edelstein, Paulos (2010)

$$\boxed{C_T = \frac{d}{d-2}\,\frac{\Gamma[d]}{\pi^{\frac{d-1}{2}}\,\Gamma\left[\frac{d-1}{2}\right]}\,\frac{\Upsilon'[\Lambda]}{(-\Lambda)^{d/2}}}$$

The dual theory of a given AdS-branch is unitary,

$$C_T > 0 \qquad \Longleftrightarrow \qquad \Upsilon'[\Lambda] > 0$$

Lovelock black holes

The black hole solution can be obtained via the ansatz

Wheeler (1986)

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\,d\Sigma^2_{\sigma,d-2}$$

where $d\Sigma_{\sigma,d-2}$ is the metric of a maximally symmetric space.

The equations of motion can be nicely rewritten as

$$\left[\frac{d}{d\log r} + (d-1)\right]\left(\sum_{k=0}^{K} c_k\, g^k\right) = 0$$

where $g(r) = \frac{\sigma - f(r)}{r^2}$, and easily solved as

$$\Upsilon[g] = \sum_{k=0}^{K} c_k\, g^k = \frac{\kappa}{r^{d-1}} \qquad \kappa = \frac{\Gamma(d/2)}{(d-2)!\,\pi^{d/2-1}}\, M$$

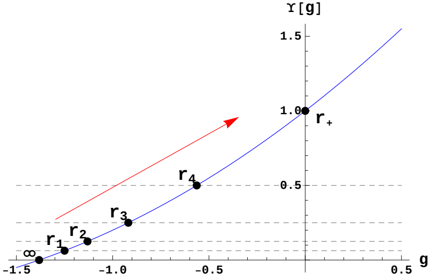
The black hole solution is implicitly given by this polynomial equation.

Lovelock black holes

Each branch, $g_i(r)$, corresponds to a monotonous part of the polynomial,

$$\Upsilon[g] = \sum_{k=0}^{K} c_k \, g^k = \frac{\kappa}{r^{d-1}}$$

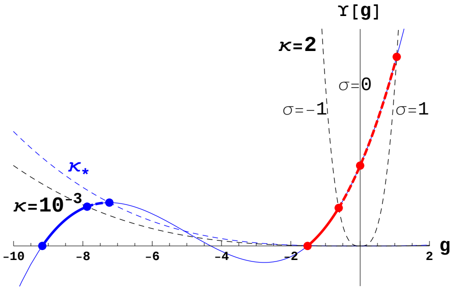
The variation of r translates the curve (y-intercept) rigidly, upwards,



This leads to K branches, $g_i(r)$, associated with each Λ_i: $g_i(r \to \infty) = \Lambda_i$

The existence of a black hole horizon requires (recall $g(r) = \frac{\sigma - f(r)}{r^2}$)

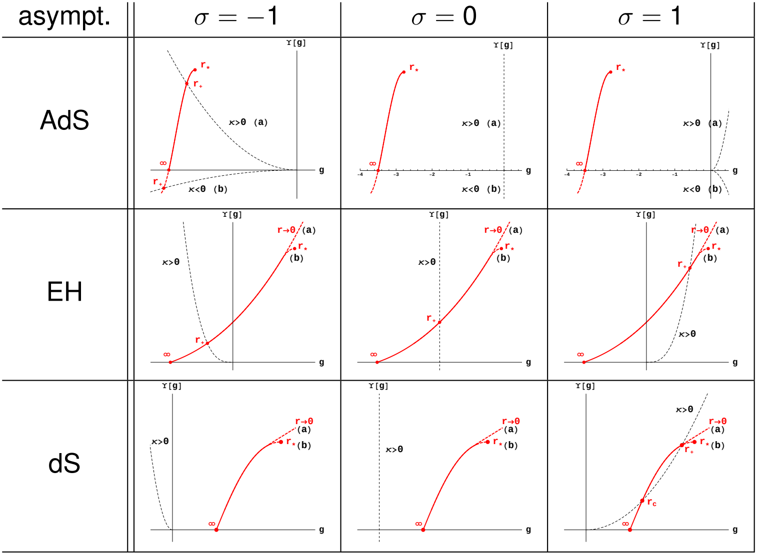
$$\Upsilon[g_+] = \frac{\kappa}{r_+^{d-1}} = \kappa \left(\sqrt{\frac{g_+}{\sigma}} \right)^{d-1} \qquad \text{since} \quad g_+ = \frac{\sigma}{r_+^2}$$



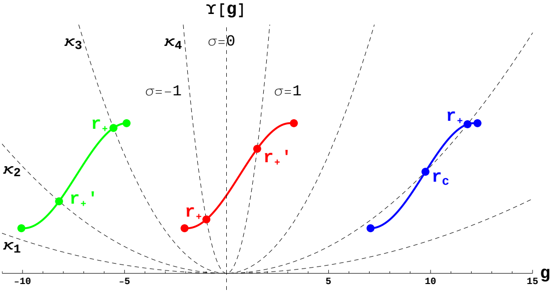
- Planar case, only the EH-branch has an event horizon.
- Non-planar case, $\sigma = \pm 1$, several branches can have the same mass or temperature $\Rightarrow$ a rich phase diagram?

Taxonomy of Lovelock black holes

Camanho, Edelstein (2011)



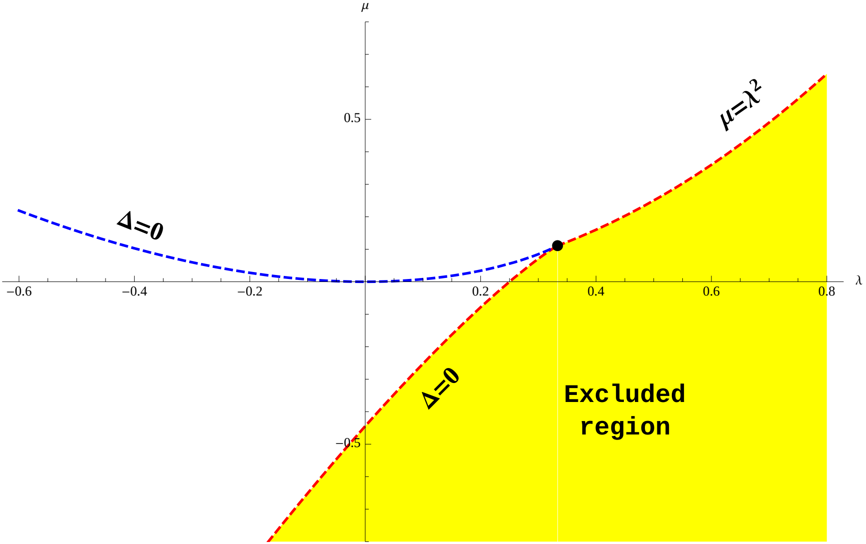
If a monotonic part of the polynomial ends at a minimum without ever touching the *g*-axis:



When the EH-branch is excluded we say that we are in the excluded region of the parameter space.

The blue branch provides a well defined spacetime for some values of the mass: both singularities hidden by the black hole and cosmological horizons.

Excluded region in cubic Lovelock theory



CFT side — Causality and positivity of the energy

Consider the operator

Hofman, Maldacena (2008)

$$\mathcal{E}(\mathbf{n}) = \lim_{r\to\infty} r^{d-2} \int_{-\infty}^{\infty} dt\, \mathbf{n}^i\, T^0{}_i(t, r\,\mathbf{n})$$

Given a state created by a local gauge invariant operator $\mathcal{O} = \epsilon_{ij}\, T_{ij}$, $\langle \mathcal{E}(\mathbf{n}) \rangle_{\mathcal{O}}$ is fully determined by the central charges in any CFT.

Since ϵ_{ij} is a symmetric and traceless polarization tensor,

$$\langle \mathcal{E}(\mathbf{n}) \rangle_{\epsilon_{ij} T_{ij}} = \mathcal{E}_0 \left[1 - \frac{2(d-1)(d-2)}{(d-3)(d-4)} \frac{\Lambda \Upsilon''[\Lambda]}{\Upsilon'[\Lambda]} \left(\frac{\mathbf{n_i}\, \epsilon^*_{il}\, \epsilon_{lj}\, \mathbf{n_j}}{\epsilon^*_{ij}\, \epsilon_{ij}} - \frac{1}{d-2} \right) \right]$$

Demanding positivity along the different channels

Buchel, Myers (2009)
Hofman (2009)
de Boer, Kulaxizi, Parnachev (2009)
Camanho, Edelstein (2009)

$$-\frac{d-2}{(d-4)} \leq -\frac{2(d-1)(d-2)}{(d-3)(d-4)} \frac{\Lambda \Upsilon''[\Lambda]}{\Upsilon'[\Lambda]} \leq d-2$$

Gravitons thrown onto shock waves must age properly

Consider a shock wave in AdS solving Lovelock's equations,

$$ds^2_{\mathrm{AdS,sw}} = ds^2_{\mathrm{AdS}} + \delta(u)\, z^{d-3}\, du^2$$

The discontinuity of P_z and the light-cone *time* v for a tensor graviton colliding the shock wave read

Hofman (2009)
Camanho, Edelstein, Paulos (2010)

$$\Delta P_z = \frac{(d-1)}{z}|P_v| \left(\frac{z}{L}\right)^2 z^{d-3} \left(1 + \frac{2(d-1)}{(d-3)(d-4)} \frac{\Lambda \Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right)$$

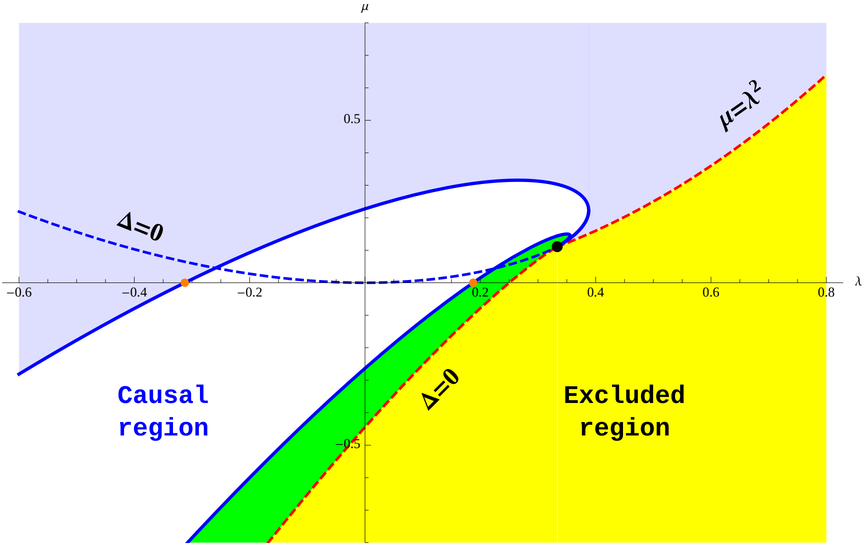
Camanho, Edelstein (2009)

$$\Delta v = \left(\frac{z}{L}\right)^2 z^{d-3} \left(1 + \frac{2(d-1)}{(d-3)(d-4)} \frac{\Lambda \Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right)$$

Thus, if the quantity in parenthesis is negative, a graviton going inside AdS bounces back, landing outside its own light-cone! For all polarizations:

$$\boxed{-\frac{d-2}{(d-4)} \leq -\frac{2(d-1)(d-2)}{(d-3)(d-4)} \frac{\Lambda \Upsilon''[\Lambda]}{\Upsilon'[\Lambda]} \leq d-2}$$

Causality restrictions in the Lovelock couplings



Causality violation and positivity of the energy

The same result can be obtained by throwing gravitons onto black holes, and seeking for superluminal *states* in the CFT. Brigante, Liu, Myers, Shenker, Yaida (2008)

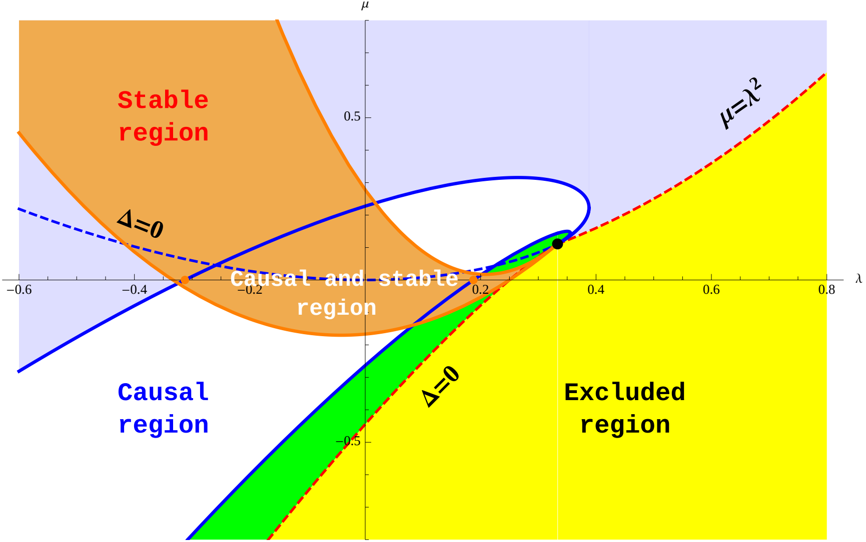
The potentials felt by high momentum gravitons in constant r slices close to the boundary, for the different helicities: de Boer, Kulaxizi, Parnachev (2009) Camanho, Edelstein (2009)

$$
\begin{aligned}
c_2^2 &\approx 1 + \frac{1}{L^2\Lambda}\left(\frac{r_+}{r}\right)^{d-1}\left[1 + \frac{2(d-1)}{(d-3)(d-4)}\frac{\Lambda\Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right] \\
c_1^2 &\approx 1 + \frac{1}{L^2\Lambda}\left(\frac{r_+}{r}\right)^{d-1}\left[1 - \frac{(d-1)}{(d-3)}\frac{\Lambda\Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right] \\
c_0^2 &\approx 1 + \frac{1}{L^2\Lambda}\left(\frac{r_+}{r}\right)^{d-1}\left[1 - \frac{2(d-1)}{(d-3)}\frac{\Lambda\Upsilon''[\Lambda]}{\Upsilon'[\Lambda]}\right]
\end{aligned}
$$

lead to the same constraints in the Lovelock couplings.

The same potentials can be expanded close to the horizon seeking for would be (plasma) instabilities, $c_k^2 < 0$ Buchel, Escobedo, Myers, Paulos, Sinha, Smolkin (2009) Camanho, Edelstein, Paulos (2010)

Restrictions in the Lovelock couplings



The shear viscosity uses the Kubo formula — Policastro, Son, Starinets (2001)

$$\eta = \frac{\sigma_{\rm abs}(\omega \to 0)}{16\pi G} = \frac{A_H}{16\pi G} \quad \Rightarrow \quad \frac{\eta}{s} = \frac{1}{4\pi}\,\frac{\hbar}{k_B}$$

This motivated the KSS bound conjecture — Kovtun, Son, Starinets (2004)

$$\frac{\eta}{s} \geq \frac{1}{4\pi}\,\frac{\hbar}{k_B}$$

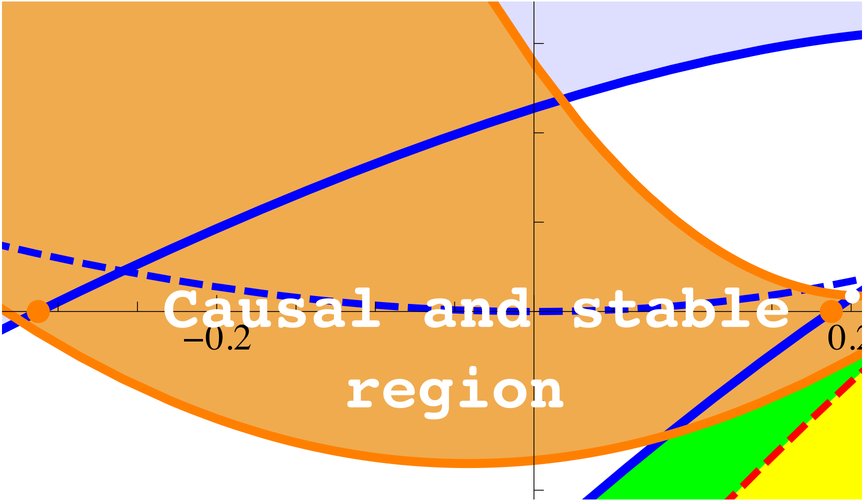
However, Lovelock terms in the gravity side lead to — Shu (2009)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 2\,\frac{d-1}{d-3}\,\lambda\right) \geq \frac{1}{4\pi}\left(1 - 2\,\frac{d-1}{d-3}\,\lambda^{\rm max}\right)$$

the ratio being reduced for $\lambda^{\rm max} > 0$

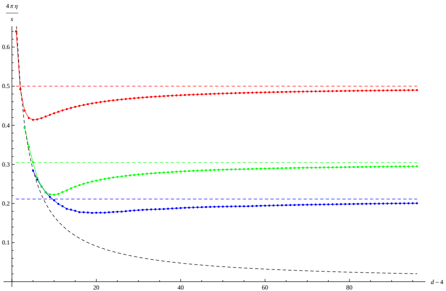
- Higher Lovelock terms **do not** contribute to η/s
- However, they **do** contribute to the lower bound of η/s! — Camanho, Edelstein, Paulos (2010)

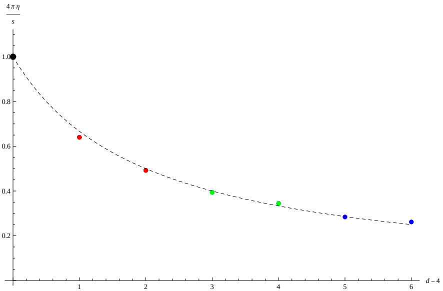
Cubic Lovelock theory allows for a lower η/s than LGB theory:



Numerical (some analytic) bounds for LGB, cubic and quartic theories.

Camanho, Edelstein, Paulos (2010)





The dashed black line in the right corresponds to the curve

$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\,\frac{2}{d-2}$$

that fits the bounds for $d < 11$ and has a nice behavior for large d.

Features of Lovelock black holes

The mass can be found through the Hamiltonian formalism Kastor, Ray, Traschen (2010)

$$M \approx {r_+}^{d-1}\, \Upsilon\left[g_+\right]$$

The horizon has a well defined temperature

$$T = \frac{f'(r_+)}{4\pi} = \frac{r_+}{4\pi}\,\left[(d-1)\,\frac{\Upsilon[g_+]}{\Upsilon'[g_+]} - 2\,g_+\right] \geq 0$$

For large r_+ (tantamount the planar case), we can approximate $M \sim T^{d-1}$

Then, $dM/dT > 0$ and the black hole is locally thermodynamically stable

The entropy reads:

$$S = \frac{A}{4}\left(1 + \sum_{k=2}^{K} k\, c_k\, \frac{d-2}{d-2k}\, {g_+}^{k-1}\right)$$

coinciding with the Euclidean on-shell action and Wald's

Myers, Simon (1988)
Jacobson, Myers (1993)

The heat capacity, $C = dM/dT$, reads

$$C \approx -{r_+}^{d-3}\,\frac{\Upsilon'\left[g_+\right]}{g_+}\,T\left[(d-2) - \frac{d-1}{2}\,\frac{\Upsilon[g_+]}{g_+\Upsilon'[g_+]}\left(1 + 2g_+\frac{\Upsilon''[g_+]}{\Upsilon'[g_+]}\right)\right]^{-1}$$

Planar black holes are locally thermodynamically stable for any mass

We can now compute the Helmholtz free energy, $F = M - T\,S$,

$$F = \frac{(d-2)\,V_{d-2}}{16\pi G_N}\,\frac{{r_+}^{d-1}}{\Upsilon'[g_+]}\sum_{k,m=0}^{K}\frac{2m-2k+1}{d-2k}\,k\,c_k\,c_m\,{g_+}^{k+m-1}$$

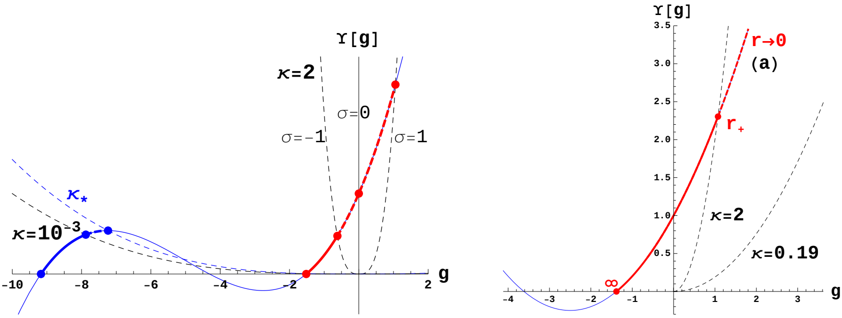
relevant to analyze the global stability of the solutions at constant T.

It has degree $2K-1$ in the numerator; that is the maximal number of zeros (for $g_+ \neq 0$) that may eventually correspond to HP-like phase transitions!

A new type of phase transition: spherical LGB black holes revisited

We would like to explore phase transitions between different branches. For this talk, we consider $\lambda > 0$ in LGB theory

Camanho, Edelstein, Giribet, Gomberoff (2012)



In the canonical ensemble, we study processes where the system undergoes a phase transition between thermal AdS_+ and a given BH_-.

How to deal with solutions that differ in the asymptotics? A likely mechanism tantamounts the *thermalon* mediated transition

Gomberoff, Henneaux, Teitelboim, Wilczek (2004)

A new type of phase transition: the two phases and the thermalon

Proviso: when $\lambda \to 0$, $\Lambda_+ \sim -1/(L^2\lambda) \to -\infty$ and one may argue that higher curvature terms cannot be neglected: think of LGB as a toy model

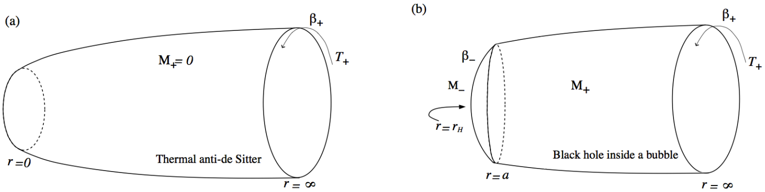


Figure: Euclidean sections for (a) empty AdS and (b) bubble hosting a black hole.

The outer region of the *bubble* asymptotes AdS space with Λ_+, while the inner region hosts a black hole with mass M_-, and Λ_-.

Solutions consisting of a spherically symmetric surface separating two regions with different vacua are known to exist.

Gravanis, Willison (2007)
Garraffo, Giribet, Gravanis, Willison (2008)

Thus: instanton transitions, $\Lambda_+ \to \Lambda_-$, via bubble nucleation.

Charmousis, Padilla (2008)

The thermalon

Each of the two (Euclidean) bulk regions read

$$ds^2 = f_\pm(r)dt_\pm^2 + \frac{dr^2}{f_\pm(r)} + r^2 d\Omega_{d-2}^2$$

At the junction surface: $r = a(\tau)$ and $t_\pm = T_\pm(\tau)$

$$ds^2 = d\tau^2 + a(\tau)^2 d\Omega_{d-2}^2 \qquad \text{as} \qquad f_\pm(a)\,\dot{T}_\pm^2 + \frac{\dot{a}^2}{f_\pm(a)} = 1$$

$a(\tau)$ being continuous across the bubble, ensures continuity of the metric.

Static configurations: same physical length of the Euclidean time circle

$$\tau = \sqrt{f_-(a)}\,T_- = \sqrt{f_+(a)}\,T_+ \qquad \Rightarrow \qquad \sqrt{f_-(a)}\,\beta_- = \sqrt{f_+(a)}\,\beta_+$$

Inner periodicity: demanding regularity at the black hole horizon.

Outer periodicity: fully determined, there is a unique free parameter.

The role of the boundary action

For *bubble* configurations, it is convenient to break the action into bulk and surface pieces, $\mathcal{M} = \mathcal{M}_- \cup \Sigma \cup \mathcal{M}_+$

$$\mathcal{I} = \left(\int_{\mathcal{M}_-} \mathcal{L}^- - \int_\Sigma \mathcal{Q}^- \right) + \left(\int_{\mathcal{M}_+} \mathcal{L}^+ + \int_\Sigma \mathcal{Q}^+ - \int_{\partial\mathcal{M}} \mathcal{Q}^+ \right)$$

Davis (2003)
Gravanis, Willison (2003)

The variation with respect to the induced vierbein on the bubble gives the junction conditions (Israel conditions of GR).

$$\widetilde{\mathcal{Q}}_{ab} = \left. \frac{\delta(\mathcal{Q}^+ - \mathcal{Q}^-)}{\delta h^{ab}} \right|_\Sigma = 0$$

The dynamics of the bubble, $(\dot{a}(\tau), a(\tau))$, is completely determined by them

$$\widetilde{\mathcal{Q}}_{ab} = 0 \qquad \Longleftrightarrow \qquad \dot{a} = \dot{a}(a; M_\pm)$$

and we may fix $M_\pm$ so that an equilibrium position exists at $a = a_\star$.

The phase transition

The canonical ensemble at temperature $1/\beta$ is defined by the path integral over all metrics which asymptote AdS identified with period β,

$$Z = \int \mathcal{D}g \, e^{-\hat{\mathcal{I}}[g]} \simeq \sum_{g_{\rm cl}} e^{-\hat{\mathcal{I}}[g_{\rm cl}]} \quad ; \qquad \hat{\mathcal{I}} = -i\mathcal{I}$$

Saddle point approximation: dominant contribution with least Euclidean action (free energy, F)

$$\hat{\mathcal{I}}_{\rm cl} \simeq -\log Z = \beta F$$

Unlike for the computation of Hawking-Page in GR, we have to consider the contribution from the boundary terms $\widetilde{\mathcal{Q}}^{\pm}$ at the bubble position $a = a_\star$.

Thermalon: Two types of contributions:

- Depending on the location of the bubble, $\hat{\mathcal{I}}_{bubble}(a_\star)$.
- The rest comes from the black hole, $\hat{\mathcal{I}}_{black\ hole} = \hat{\mathcal{I}} - \hat{\mathcal{I}}_{bubble} = \beta_- M_- - S$

The junction conditions and thermodynamic consistency

Once the junction conditions are imposed for ($\dot{a} = \ddot{a} = 0, a = a_\star$),

$$\hat{\mathcal{I}}_{bubble} = \beta_+ M_+ - \beta_- M_-$$

Needed to preserve the thermodynamic interpretation,

$$\hat{\mathcal{I}} = \beta_+ M_+ - S$$

The bubble contributes as mass but does not add to the entropy.

Hamiltonian approach: contributions from total charges of the solution.
Junction conditions $\Leftrightarrow$ continuity of canonical momenta. Bañados, Teitelboim, Zanelli (1994)

The junction conditions also preserve the first law of thermodynamics

$$\beta_+ dM_+ = \beta_- dM_- = dS$$

It holds for both the inner black hole and the thermalon.

Global thermodynamic stability: sign of the free energy

There is a critical temperature, $T_c(\lambda)$, above which F becomes negative triggering the phase transition.

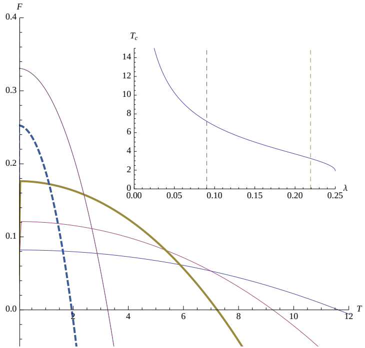


Figure: Free energy versus temperature in $d = 5$ for $\lambda = 0.04, 0.06, 0.09$ (positivity bound), 0.219 (maximal $F(T = 0)$), and $\lambda \to 1/4$ (from right to left).

$T_c(\lambda)$ is monotonically decreasing $\Rightarrow$ increasingly unlikely the more we come closer to the EH – *classical* – limit, $\lambda \to 0$.

How does the bubble evolve? From the (Lorentzian) junction conditions:

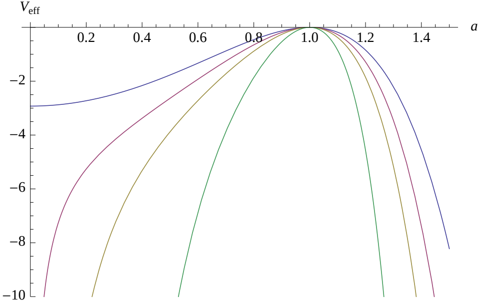


Figure: Bubble potential for $\lambda = 0.1$ and $d = 5, 6, 7, 10$.

The bubble may expand reaching the boundary at finite proper time thus changing the asymptotics and the charges:

$$\Lambda_+ \longrightarrow \Lambda_- \qquad \text{and} \qquad (M_+, T_+) \longrightarrow (M_-, T_-)$$

Final remarks

- **Lovelock theory** is a useful playground for AdS/CFT.
- A novel mechanism for phase transitions in higher curvature gravity: mimicking the thermalon configuration, **a bubble pops out with a black hole in its interior**.
- Thermodynamically preferred above $T_c(\lambda)$, a **generalized HP phase transition** is triggered **for the higher-curvature branches**.
- The bubble dynamically **modifies the cosmological constant**, driving the system **towards the EH-branch**.
- Branches of asymptotically (A)dS solutions should be interpreted as **different phases of the dual field theory**.
- **Confinement/deconfinement** transition between strongly coupled CFTs involving **an effective change in the 't Hooft coupling**.
- It deserves further exploration.

Thank you for your attention