The Road to Flat Space Holography.

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EMPG Seminar
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References

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- Relatively much less understood about flat space holography.
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- Symmetries will be the principle ingredient of our proposal. We will make use of the BMS group.
- On the CFT side, this limit induces a contraction of the relativistic CFT and structures studied in the context of non-relativistic CFTs appear.

Plan of the talk

Asymptotic Symmetries and Flat-spacetimes

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Asymptotic Symmetries and Flat-spacetimes

Non-Relativistic Symmetries

The BMS/GCA Correspondence

Making sense of all that: the bulk side.

Making sense of all that: the CFT side.

The Curious Case of the Flat BTZ

Outline

Asymptotic Symmetries

$$\text{Asymptotic Symmetry Group} = \frac{\text{Allowed Diffeomorphisms}}{\text{Trivial Diffeomorphisms}}$$

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- Trivial Diffeos: Conserved charges associated with these diffeos are zero.
- Generators of ASG: Diffeos allowed by the boundary conditions but decay just enough at the boundary to allow non-zero charges.

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- In a quantum theory, the states form representations of ASG.
- Often, ASG→ isometry of the vacuum: $\text{ASG}(\text{AdS}_{d+2}) = \text{SO}(d+1, 2)$. (for $d>1$.)
- Famous exception: $\text{ASG}(\text{AdS}_3) = \text{Virasoro} \otimes \text{Virasoro}$. Brown, Henneaux 1986.
- States of Quantum Gravity in AdS_3 would be characterised by the underlying infinite dimensional conformal algebra.
- Another interesting example: Near Horizon Extreme Kerr. Bardeen, Horowitz 1999; Guica, Hartman, Song, Strominger 2008.

Asymptotic Symmetries of Flat Space

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- Non-trivial ASG for asymptotically flat spacetimes at null infinity in 3d and 4d.
- BMS_3:

$$[J_m, J_n] = (m-n)J_{m+n}, \quad [J_m, P_n] = (m-n)P_{m+n}, \quad [P_m, P_n] = 0. \tag{1}$$

J_m = Global Conformal Generators of S^1.
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- Further Extension: Can extend J_m to a Virasoro algebra by not requiring generators to be globally defined. Can compute central extensions.

$$c_{JJ} = 0, \quad c_{JP} = \frac{3}{G}, \quad c_{PP} = 0. \tag{2}$$

  Barnich-Compere 2006.

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$$[L_m, L_n] = (m-n)L_{m+n}, \quad [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}, \quad [L_m, \bar{L}_n] = 0,$$
$$[L_k, T_{m,n}] = (\frac{k+1}{2} - m)T_{m+k,n}, \quad [\bar{L}_k, T_{m,n}] = (\frac{k+1}{2} - n)T_{m,n+k}. \tag{3}$$

$L_m, \bar{L}_m$ = Global Conformal Generators of S^2.
$T_{m,n}$ = "Super" translations → Infinite dimensional.
(m,n = 0, ±1): Poincare group in 4d.
Bondi, van der Burg, Metzner; Sachs 1962.

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- Attempts at BMS$_3$/CFT$_1$ and BMS$_4$/CFT$_2$.
  Barnich, Troessaert 2010.

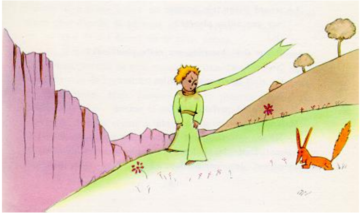
Outline

Inönü-Wigner Contractions: A Simple Example

$SO(3)$ maps the surface of the sphere (S^2) embedded in R_3 to itself.

- Equation for S^2: $x_1^2 + x_2^2 + x_3^2 = R^2$.
- Infinitesimal generators: $X_{ij} = x_i\partial_j - x_j\partial_i$
- Algebra: $[X_{ij}, X_{rs}] = X_{is}\delta_{jr} + X_{jr}\delta_{is} - X_{ir}\delta_{js} - X_{js}\delta_{ir}$

Inönü-Wigner Contractions: A Simple Example ...



Take the limit $R \to \infty$. Look at the north pole: $x_{1,2} = 0$ and $x_3 = R$.

$$Y_{12} = \lim_{R\to\infty} X_{12} = x_1\partial_2 - x_2\partial_1, \quad P_i = \lim_{R\to\infty} \frac{1}{R} X_{i,3} = \lim_{R\to\infty} \frac{1}{R}(x_i\partial_3 - x_3\partial_i) \to -\partial_i$$

Redefined algebra: $\boxed{[Y_{12}, P_i] = P_1\delta_{2i} - P_2\delta_{1i}, \quad [P_1, P_2] = 0} \to$ ISO(2).

Will use this to construct non-relativistic limit of conformal algebra.

Non-Relativistic Contraction: Finite GCA

Relativistic CFT generators: $\{\ J^{ij}(J_0^a); \{H, D, K^0\}(L_{-1,0,1}); \{P^i, B^i, K^i\}(M^i_{-1,0,1})\ \}$ (Poincare sub-algebra denoted in blue).

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- Non-relativistic limit: Scale (with $\epsilon \to 0$): $t \to t, \;\; x_i \to \epsilon x_i$. Equivalent to $v_i \sim \epsilon$ $(c = 1)$.
- Poincare sub-algebra $\to$ Galilean algebra.
- Relativistic Conformal Algebra $\to$ Galilean Conformal Algebra (GCA).

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The finite dimensional GCA (with $m, n = 0, \pm 1$):

$$[L_m, L_n] = (m-n)L_{m+n}, \quad [L_m, M^i_n] = (m-n)M^i_{m+n}, \quad [M^i_n, M^j_m] = 0,$$
$$[J^{ij}, L_n] = 0, \quad [J^{ij}, M^k_m] = -(M^i_m \delta^{jk} - M^j_m \delta^{ik}).$$

AB, R. Gopakumar 2009.

Infinite GCA

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- For $n = 0, \pm 1; \ell = 0$, vector fields that generate GCA are

$$L_n = -(n+1)t^n x_i \partial_i - t^{n+1}\partial_t, \quad M_n^i = t^{n+1}\partial_i, \quad J_\ell^a \equiv J_\ell^{ij} = -t^\ell (x_i \partial_j - x_j \partial_i).$$

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- Define these vector fields for *arbitrary* integer n.
- We get a Virasoro Kac-Moody like algebra

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- Natural infinite extension of finite GCA *for any dimension of spacetime*.
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- Natural infinite extension of finite GCA *for any dimension of spacetime*.
- Enhancement of symmetries in the non-relativistic limit.
- Can build representations, construct correlation-functions from symmetry algebra.

AB, I. Mandal 2009.

Focussing on D=2

AB, Gopakumar, Mandal, Miwa 2009.

- GCA in 2d:

$$[L_m, L_n] = (m-n)L_{m+n} + C_1\delta_{m+n,0}m(m^2-1),$$
$$[L_m, M_n] = (m-n)M_{m+n} + C_2\delta_{m+n,0}m(m^2-1)$$
$$[M_m, M_n] = 0.$$

- Central charges added to the classical algebra described previously. C_2 is special to 2d.
- D=2 is special $\rightarrow$ Relativistic Conformal Algebra is infinite dimensional.
- Natural question: are these two infinite algebras related in any (simple) way?
- Answer: Yes!
- The infinite GCA in 2d can be related to the contraction of linear combinations of two copies 2d Virasoro in a way we will describe later.

Outline

BMS/GCA: Connecting infinite algebras.

A Bagchi 2010.

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Red Herring?

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- Start from 3d Relativistic Conformal theory (t, x, y). Anisotropic contraction: $(t, y) \rightarrow (t, y)$; $x \rightarrow \epsilon x$. : "Semi"-GCA
- Re-write finite algebra to form algebra with $SL(2, R) \otimes SL(2, R)$.
- As before, can lift this to an infinite algebra and this is *exactly* BMS_4.

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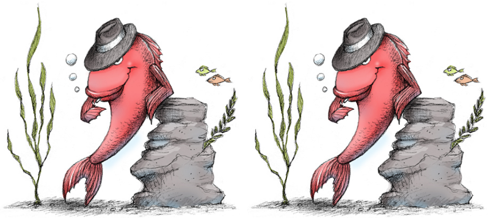
- So, $\boxed{BMS_4 = s\ GCA_3}$.

BMS/GCA in d=4

- Two red herrings?



- Unlikely. :-) ... let's take this seriously!

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Flat-Space limit of AdS_3

- Asymptotic symmetry group of AdS_3 = Vir $\otimes$ Vir.
- Asymptotic symmetry algebra: $[\mathcal{L}_n, \mathcal{L}_m] = (n-m)\mathcal{L}_{n+m} + \frac{c}{12}\delta_{n+m,0}(n^3 - n)$ and similarly for $\bar{\mathcal{L}}_n$.

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- Flat space arises as a limit of AdS when the AdS radius is taken to infinity. This is a contraction from the algebraic sense.
- BMS algebra is generated by a simple contraction of the linear combinations of $\mathcal{L}_n, \bar{\mathcal{L}}_n$.

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \frac{1}{\ell}(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}) \tag{4}$$

where ℓ is the AdS radius.

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + c_{LL}\delta_{n+m,0}(n^3 - n). \\ [L_n, M_m] &= (n-m)M_{n+m} + c_{LM}\delta_{n+m,0}(n^3 - n). \end{aligned} \tag{5}$$

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- Naturally generates the central charges: $c_{LM} = \frac{1}{\ell}(c+\bar{c}) = \frac{3}{G}$ and $c_{LL} = c - \bar{c} = 0$ as $c = \bar{c} = \frac{3\ell}{2G}$.

Flat-Space limit: Representation and Contraction

AB, R Fareghbal 2012.

- We will see that the flat-space limit of $\ell \to \infty$ *naturally induces* the contraction $\{t \to \epsilon t, x \to x\}$ on the conformal field theory on the boundary.
- In this new contraction, the boundary CFT is again reduced to the 2d GCA, but has a different spacetime interpretation from the previous non-relativistic contraction.

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- For this we will look at the Killing vectors of global AdS$_3$:

$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) d\tau^2 + dr^2 \left(1 + \frac{r^2}{\ell^2}\right)^{-1} + r^2 d\phi^2. \tag{6}$$

- We will go near the boundary of AdS$_3$ ($\ell/r \to 0$ at finite but large r) . We would then take the flat-space limit $\ell \to \infty$.
- AdS is best viewed as a box. To look at the symmetries, we first go to the wall of the box and then push the wall out to infinity.

Killing Vectors: The Horror Slide

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Step1: Killing vectors of Global AdS_3:

$$
\begin{aligned}
J_{01} &= \ell\partial_\tau\,, \quad J_{02} = \sqrt{\ell^2+r^2}\cos\phi\cos\frac{\tau}{\ell}\,\partial_r - \frac{r\ell}{\sqrt{\ell^2+r^2}}\cos\phi\sin\frac{\tau}{\ell}\,\partial_\tau - \frac{\sqrt{\ell^2+r^2}}{r}\sin\phi\cos\frac{\tau}{\ell}\,\partial_\phi \\
J_{03} &= \sqrt{\ell^2+r^2}\sin\phi\cos\frac{\tau}{\ell}\,\partial_r - \frac{r\ell}{\sqrt{\ell^2+r^2}}\sin\phi\sin\frac{\tau}{\ell}\,\partial_\tau + \frac{\sqrt{\ell^2+r^2}}{r}\cos\phi\cos\frac{\tau}{\ell}\,\partial_\phi \\
J_{12} &= \sqrt{\ell^2+r^2}\cos\phi\sin\frac{\tau}{\ell}\,\partial_r + \frac{r\ell}{\sqrt{\ell^2+r^2}}\cos\phi\cos\frac{\tau}{\ell}\,\partial_\tau - \frac{\sqrt{\ell^2+r^2}}{r}\sin\phi\sin\frac{\tau}{\ell}\,\partial_\phi \\
J_{13} &= \sqrt{\ell^2+r^2}\sin\phi\sin\frac{\tau}{\ell}\,\partial_r + \frac{r\ell}{\sqrt{\ell^2+r^2}}\sin\phi\cos\frac{\tau}{\ell}\,\partial_\tau + \frac{\sqrt{\ell^2+r^2}}{r}\cos\phi\sin\frac{\tau}{\ell}\,\partial_\phi\,, \quad J_{23} = -\partial_\phi
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\end{aligned}$$

Step 2: Going near the boundary. ($\ell/r \to 0$.)

$$J_1 = \ell\partial_\tau\,, \quad J_2 = r\cos\phi\cos\frac{\tau}{\ell}\,\partial_r - \ell\cos\phi\sin\frac{\tau}{\ell}\,\partial_\tau - \sin\phi\cos\frac{\tau}{\ell}\,\partial_\phi\,, \quad J_3 = r\sin\phi\cos\frac{\tau}{\ell}\,\partial_r - \ell\sin\phi\sin\frac{\tau}{\ell}\,\partial_\tau + \cos\phi\cos\frac{\tau}{\ell}\,\partial_\phi$$

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Killing Vectors: The Horror Slide

Step1: Killing vectors of Global AdS_3:

$$J_{01} = \ell\partial_\tau\,, \quad J_{02} = \sqrt{\ell^2+r^2}\cos\phi\cos\frac{\tau}{\ell}\,\partial_r - \frac{r\ell}{\sqrt{\ell^2+r^2}}\cos\phi\sin\frac{\tau}{\ell}\,\partial_\tau - \frac{\sqrt{\ell^2+r^2}}{r}\sin\phi\cos\frac{\tau}{\ell}\,\partial_\phi$$

$$J_{03} = \sqrt{\ell^2+r^2}\sin\phi\cos\frac{\tau}{\ell}\,\partial_r - \frac{r\ell}{\sqrt{\ell^2+r^2}}\sin\phi\sin\frac{\tau}{\ell}\,\partial_\tau + \frac{\sqrt{\ell^2+r^2}}{r}\cos\phi\cos\frac{\tau}{\ell}\,\partial_\phi$$

$$J_{12} = \sqrt{\ell^2+r^2}\cos\phi\sin\frac{\tau}{\ell}\,\partial_r + \frac{r\ell}{\sqrt{\ell^2+r^2}}\cos\phi\cos\frac{\tau}{\ell}\,\partial_\tau - \frac{\sqrt{\ell^2+r^2}}{r}\sin\phi\sin\frac{\tau}{\ell}\,\partial_\phi$$

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Step 3: Notice τ always comes as $\frac{\tau}{\ell}$ in all expressions. Absorb radius of AdS by $t = \frac{\tau}{\ell}$.

$$J_1 = \partial_t\,, \quad J_2 = r\cos\phi\cos t\,\partial_r - \cos\phi\sin t\,\partial_t - \sin\phi\cos t\,\partial_\phi\,, \quad J_3 = r\sin\phi\cos t\,\partial_r - \sin\phi\sin t\,\partial_t + \cos\phi\cos t\,\partial_\phi$$

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Killing Vectors: The Flat Limit

Step 4: Observe: $\boxed{\ell \to \infty \Rightarrow t \to 0}$.

Redefine: $L_0 = -iJ_6, \;\; L_{\pm 1} = \pm J_2 + iJ_3, \quad M_0 = iJ_1, \;\; M_{\pm 1} = \mp J_5 + iJ_4.$

Contracted Generators: $\boxed{L_n = ie^{in\phi}(\partial_\phi + int\partial_t - inr\partial_r), \;\; M_n = ie^{in\phi}\partial_t}$ for $n = -1, 0, 1.$

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Step 5: As before define this for any integer $n \to$ an infinite lift to BMS_3 algebra.

$$[L_n, L_m] = (n-m)L_{n+m}, \quad [L_n, M_m] = (n-m)M_{n+m}, \quad [M_n, M_m] = 0.$$

Step 6: On the boundary $r \to \infty$, reduces to generators of GCA_2 defined on cylinder.

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So: This gives a clear picture of the space-time interpretation of the BMS/GCA correspondence.

Flat-Space limit: Where is the CFT?

- Where does the contracted CFT live?

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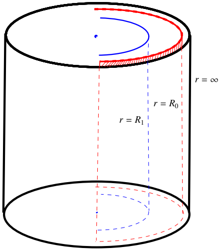
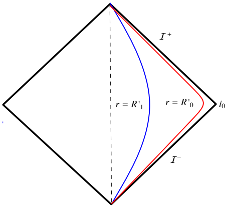


(a) : Penrose Diagram for AdS_3



(b) : Penrose Diagram for $3\,d$ Minkowski

- On the full null conformal boundary of flatspace: $\{\mathcal{I}^+, \mathcal{I}^-\}$.

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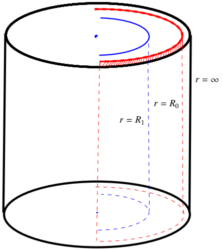

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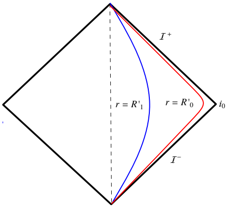

(b) : Penrose Diagram for 3 d Minkowski

- On the full null conformal boundary of flatspace: $\{\mathcal{I}^+, \mathcal{I}^-\}$.
- Can repeat the same analysis to get exactly similar results for BMS_4.

Contraction of Brown-Henneaux Asymptotic Symmetries

AB, R Fareghbal 2012.

- Brown and Henneaux boundary conditions in global coordinates:

$$\begin{pmatrix} h_{\tau\tau} = \mathcal{O}(1) & h_{\tau r} = \mathcal{O}(1/r^3) & h_{\tau\phi} = \mathcal{O}(1) \\ & h_{rr} = \mathcal{O}(1/r^4) & h_{r\phi} = \mathcal{O}(1/r^3) \\ & & h_{\phi\phi} = \mathcal{O}(1) \end{pmatrix}. \tag{8}$$

- The generators of the asymptotic symmetry algebra are given by

$$\mathcal{L}_n^{\pm} = \frac{i}{2} \exp\left(in(\frac{\tau}{\ell} \pm \phi)\right) \left[\ell\partial_\tau - inr\partial_r \pm \partial_\phi\right] \tag{9}$$

- Redefining as before $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \;\; M_n = \frac{1}{\ell}(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$, in the limit $\ell \to \infty$:

$$L_n = ie^{in\phi}\left(\partial_\phi + int\partial_t - inr\partial_r\right) \quad M_n = ie^{in\phi}\partial_t. \tag{10}$$

- We have reproduced the *exact forms of the asymptotic Killing vectors* which we had obtained by observation previously.

Outline

Representation of 2d-GCA

Contraction on cylinder.

- Relativistic Generators: $\mathcal{L}_n = ie^{in\omega}\partial_\omega, \quad \bar{\mathcal{L}}_n = ie^{in\bar{\omega}}\partial_{\bar{\omega}}.$
- GCA generators: $L_n = ie^{inx}(\partial_x + int\partial_t), \quad M_n = ie^{inx}\partial_t.$
- GCA Central terms: $c_{LL} = C_1 = \frac{c-\bar{c}}{12}, \; c_{LM} = C_2 = \epsilon\frac{c+\bar{c}}{12}, \; c_{MM} = 0.$

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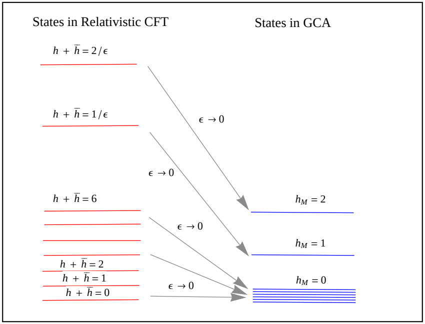
Representations of GCA

- States labeled under $\{L_0, M_0\}$.

$$L_0|h_L, h_M\rangle = h_L|h_L, h_M\rangle, \quad M_0|h_L, h_M\rangle = h_M|h_L, h_M\rangle \tag{11}$$

  with $h_L = h - \bar{h}$, $h_M = \epsilon(h + \bar{h})$.
- Demanding spectrum to be bounded from below, usual notion of primary states in the algebra: states that are annihilated by all L_n, M_n with $n > 0$.
- GCA modules built out of primaries by acting on them with raising operators L_{-n}, M_{-n} with $n > 0$.

Correlation functions



Correlation Functions:

- The limit from the relativistic correlation functions yield interesting structures.
- There is interesting dynamics in the $h_M = 0$ subsector.
- Assume there is a small parameter $\hat{\epsilon}$ which lifts the degeneracy of the ground state and distinguishes between the states from the parent theory which collapse to $h_M = 0$.

Correlation functions...

Correlation Functions:

- Take a co-ordinated limit, so that we are near $h_M = 0$ as we contract from the Virasoros. $h_M = \hat{\epsilon}\Delta$ and $\epsilon, \hat{\epsilon}$ go to zero at the same rate.
- Two point function of two primary operators is given by

$$G^{(2)} = \tilde{C}(1 - \cos x)^{-\Delta} \quad (12)$$

- If we also focus on a sector of high spins $h_L = \frac{\xi}{\hat{\epsilon}}$, we get further structure:

$$G^{(2)} = \tilde{C}(1 - \cos x)^{-\Delta} \exp\left(\xi t \cot(\frac{x}{2})\right) \quad (13)$$

Summary so far

- $\mathrm{BMS}_3 = \mathrm{GCA}_2$. ($\mathrm{BMS}_4 = \mathrm{sGCA}_3$.)

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- Linear combination of AdS_3 ASG Vir $\otimes$ Vir reduces to BMS_3 in the $\ell \to \infty$ limit.
- Algebra of contracted Killing vectors near the boundary can be lifted to infinite dimensional BMS.
- In 3d, contraction of Brown-Henneaux generators reproduce above Killing vectors.

Summary so far

- $\text{BMS}_3 = \text{GCA}_2$. ($\text{BMS}_4 = \text{sGCA}_3$.)

- Linear combination of AdS_3 ASG Vir $\otimes$ Vir reduces to BMS_3 in the $\ell \to \infty$ limit.
- Algebra of contracted Killing vectors near the boundary can be lifted to infinite dimensional BMS.
- In 3d, contraction of Brown-Henneaux generators reproduce above Killing vectors.

- Contraction on CFT $t \to \epsilon t, x \to x$ to give GCA_2 is induced by $\ell \to \infty$ on the bulk.
- Flat space quantum gravity in 3d would be given by representations of GCA_2.
- Constructed representations and correlation functions.
- Interesting ground state dynamics.

Outline

"Flat" BTZ

AB, Detournay, Fareghbal, Simon 1208.4372.

Barnich et al 1204.3288

- Can obviously take the same limit on the BTZ black hole in AdS_3.
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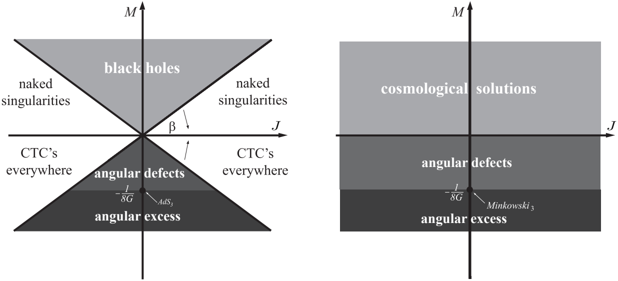
[Figure: from Barnich et al 1204.3288]



Revisiting the BTZ Black Hole.

BTZ Basics: The bulk side.

- The BTZ black hole solution to AdS_3 is given by

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2\ell^2}dt^2 + \frac{r^2\ell^2}{(r^2 - r_+^2)(r^2 - r_-^2)}dr^2 + r^2\left(d\phi + \frac{r_+ r_-}{\ell r^2}dt\right)^2.$$

- Here $r_\pm = \sqrt{2G\ell(\ell m + j)} \pm \sqrt{2G\ell(\ell m - j)}$;
- m and j are the mass and angular momentum of the black hole.
- Bekenstein-Hawking entropy of the BTZ is a quarter of the area of the event horizon:

$$S_{BH} = \frac{\pi r_+}{2G}. \tag{14}$$

Revisiting the BTZ Black Hole ...

BTZ Basics: The dual side.

- The central charges of 2-dimensional dual CFT are $c = \bar{c} = \frac{3\ell}{2G}$.
- The eigenvalues of $\mathcal{L}_0$ and $\bar{\mathcal{L}}_0$ are respectively

$$h = \frac{1}{2}(\ell m + j) + \frac{c}{24}, \qquad \bar{h} = \frac{1}{2}(\ell m - j) + \frac{\bar{c}}{24} \tag{15}$$

- Magic of 2d CFT: Can count states just by the infinite symmetry algebra without knowing the exact details of the dual.
- Cardy's formula:

$$S_{CFT} = 2\pi\left(\sqrt{\frac{c\,h}{6}} + \sqrt{\frac{\bar{c}\,\bar{h}}{6}}\right). \tag{16}$$

- Plugging back, we get

$$S_{BH} = S_{CFT} \tag{17}$$

Flattening the BTZ

- $\ell \to \infty$:

$$r_{\pm} = \ell\sqrt{2Gm}\Big[(1+\frac{j}{\ell m})^{1/2} \pm (1-\frac{j}{\ell m})^{1/2}\Big]$$

$$r_{+} \to \ell\sqrt{2Gm}, \quad r_{-} \to r_0 = \sqrt{\frac{2G}{m}}j$$

- r_+ is pushed out to infinity. One is left with the inside.
- Radial coordinate r becomes time-like and the time coordinate t becomes spatial.
- Cosmology!
- r_- survives the limit. Cosmological Horizon!
- Studied earlier by Cornalba and Costa as an orbifold of flat-space.
- Can understand this by the fact that BTZ is an orbifold of AdS_3 and by following the limit carefully.

Entropy and the Ring

- After the limit, redefining $\tau = r$, $\tau_0 = r_0$, $\psi = \phi + \frac{2m}{j} t$

$$\text{Metric:} \quad ds^2 = -\frac{\tau^2 d\tau^2}{8Gm(\tau^2 - \tau_0^2)} + (\tau^2 - \tau_0^2) d\phi^2 + \tau_0^2 d\psi^2 \qquad (18)$$

- Define new coordinate v as $dv = d\phi + \frac{\tau d\tau}{\sqrt{8Gm}(\tau^2 - \tau_0^2)}$

$$\text{Metric:} \quad ds^2 = (\tau^2 - \tau_0^2) dv^2 - \frac{2\tau dv d\tau}{\sqrt{8Gm}} + \tau_0^2 d\psi^2 \qquad (19)$$

- $\tau = \tau_0$ is a null hypersurface. Killing vector $\xi = \partial_v$ is normal to it.
- Therefore $\tau = \tau_0$ is a Killing horizon $\Rightarrow$ Cosmological Horizon.
- Using Bekenstein-Hawking area law to define the entropy of our cosmological horizon we get

$$\boxed{S_{CH} = \frac{\pi \tau_0}{2G} = \frac{\pi j}{\sqrt{2Gm}}} \qquad (20)$$

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- Can write a partition function for the GCA.
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$$\boxed{S_{GCA} = \ln d(h_L, h_M) = \pi\left(C_1\sqrt{\frac{2h_M}{C_2}} + h_L\sqrt{\frac{2C_2}{h_M}}\right).} \tag{21}$$

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- For our cosmological ring solution,

$$h_L = j + \frac{C_1}{2}, \quad h_M = m + \frac{C_2}{2}; \quad C_1 = 0, \quad C_2 = \frac{1}{4G} \tag{22}$$

Plugging this back, in the limit of large charges, we get:

$$S_{GCA} = \frac{\pi j}{\sqrt{2Gm}} \tag{23}$$

- So we see that we have a perfect matching of the entropies calculated in the two different ways.

$$\boxed{S_{GCA} = S_{CR}} \tag{24}$$

Conclusions

- Main Message: Flat space holography can be obtained as a limit of usual AdS/CFT.
- At least in d=3, the limit is relatively straight-forward.
- Many interesting calculations like computation of the entropy of cosmological horizons from the field theory side can be carried out. This should be looked at as the flat-space version of Strominger's derivation of the BTZ entropy.
- One can look to address many questions in the 3d case just by carefully looking at the limit of AdS_3/CFT_2.
- The 4d bulk is even more interesting as an infinite symmetry arises there which is not present in AdS_4.

Important Questions for the Future

Black Hole Entropy

- All non-extremal black holes have "near-horizon" geometries which are 2d-Rindler spacetimes.
- Our cosmological solution can be reinterpreted as a 2d Rindler spacetime in a certain patch.
- A road to the elusive Scharzschild entropy?

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- Asymptotically flat 4d black holes: understanding in terms of BMS_4?
- What does this asymptotic symmetry have to do with say NHEK?

Important Questions for the Future

Correlation functions and Mellin amplitudes

- BMS is the symmetry group at $\mathcal{I}^{\pm}$. Hence it should be the symmetry structure important for the S-matrix.
- Recent advances in S-matrix theory in flat-spaces through the language of Mellin amplitudes.
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Many other potentially very interesting avenues of future research.

Thank you !