Non-AdS holography in 3D higher spin gravity

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FWF

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Holographic algorithm from gravity point of view

Universal recipe & Outline of the talk:

1. Identify bulk theory and variational principle

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Goal of this talk:

Apply algorithm above to non-AdS holography in 3D higher spin gravity

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Why not blithely stick to AdS holography?

Non-AdS holography is of interest because...

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Box with simple non-AdS backgrounds:

- Minkowski
- dS
- AdS$_{D-n} \times S^n$
- Schrödinger
- Lifshitz
- warped AdS

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> We have a lot of technical control on both sides of possible holographic correspondences

Note: 2D is even simpler, but for many aspects too simple:

- No graviton excitations (even off-shell)
- No analogue of surface area of event horizon
- Pure Einstein gravity has no equations of motion

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- (Toy) models for consistent theories of quantum gravity, particularly in three dimensions (e.g. Henneaux/Rey, Campoleoni/Fredenhagen/Pfenninger/Theisen '10 or Bagchi/Lal/Saha/Sahoo '11 or Castro/Gaberdiel/Hartman/Maloney/Volpato '12)

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- It is there...

> Focus in rest of talk on non-AdS holography in 3-dimensional higher spin gravity

Holographic algorithm from gravity point of view

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Bulk theory and variational principle

Chern–Simons theory with some gauge algebra that contains $sl(2) \times sl(2)$

$$I = I_{\text{CS}}[A] - I_{\text{CS}}[\bar{A}]$$

with

$$I_{\text{CS}}[A] = \frac{k}{4\pi} \int_{\mathcal{M}} \mathsf{Tr}(A \wedge \mathrm{d}A + \tfrac{2}{3} A \wedge A \wedge A) + B[A]$$

and

$$B[A] = \frac{k}{4\pi} \int_{\partial\mathcal{M}} \mathsf{Tr}(A_+ \, \mathrm{d}x^+ \, A_- \, \mathrm{d}x^-)$$

Gauge invariant if infinitesimal gauge parameter obeys boundary condition

$$\partial_- \epsilon \big|_{\partial\mathcal{M}} = 0$$

Variational principle consistent for

$$\delta A_- \big|_{\partial\mathcal{M}} = 0 \qquad \text{or} \qquad A_+ \big|_{\partial\mathcal{M}} = 0$$

Bar-sector works similarly, exchanging $\pm$

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Background and fluctuations

Take suitable group element b (often: $b = e^{\rho L_0}$) and make Ansatz for connection

$$A = b^{-1}\left(\hat{a}^{(0)} + a^{(0)} + a^{(1)}\right) b$$

- $\hat{a}^{(0)} \sim \mathcal{O}(1)$: determines asymptotic background
- $a^{(0)} \sim \mathcal{O}(1)$: determines state-dependent fluctuations
- $a^{(1)} \sim o(1)$: sub-leading fluctuations

Bar-sector is analog

Boundary-condition preserving gauge transformations generated by ϵ

$$\epsilon = b^{-1}\left(\epsilon^{(0)} + \epsilon^{(1)}\right) b$$

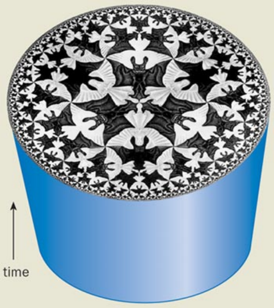
with $\epsilon^{(0)} \sim \mathcal{O}(1)$ (subject to constraints) and $\epsilon^{(1)} \sim o(1)$

Metric is then determined from

$$g_{\mu\nu} = \frac{1}{2}\,\mathrm{Tr}\left[(A - \bar{A})_\mu (A - \bar{A})_\nu\right]$$

Example: Lobachevsky holography

Lobachevsky plane times time:



Lobachevsky background ($x^+ = t$, $x^- = \varphi$):

$$\mathrm{d}s^2 = \mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2\!\rho \; \mathrm{d}\varphi^2$$

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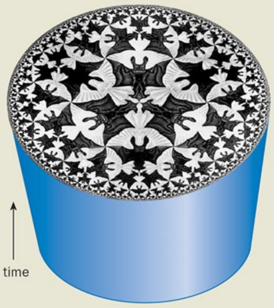

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Connections in n-p embedding of spin-3 gravity:

$$\begin{aligned} A_\rho &= L_0 & \bar{A}_\rho &= -L_0 \\ A_\varphi &= -\frac{1}{4}\, e^\rho L_1 & \bar{A}_\varphi &= -e^\rho L_{-1} \\ A_t &= 0 & \bar{A}_t &= \sqrt{3} S \end{aligned}$$

Indeed $\hat{a}^{(0)}$ is ρ-independent for $b = e^{\rho L_0}$

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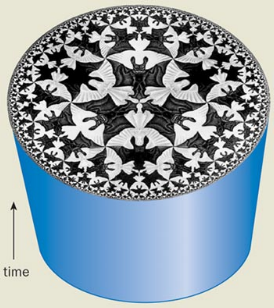

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Fluctuations:

$$a_\varphi^{(0)} = \frac{2\pi}{k}\left(\tfrac{3}{2}\mathcal{W}_0(\varphi)S + \mathcal{W}^+_{\frac12}(\varphi)\psi^+_{-\frac12} - \mathcal{W}^-_{\frac12}(\varphi)\psi^-_{-\frac12} - \mathcal{L}(\varphi)L_{-1}\right)$$

$$a_\mu^{(1)} = \mathcal{O}(e^{-2\rho})$$

Bar-sector is similar

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Background independent result:

$$\delta Q[\epsilon] = \frac{k}{2\pi} \oint \mathrm{Tr}\,(\epsilon^{(0)}\,\delta a_\varphi^{(0)}\,\mathrm{d}\varphi)$$

- Manifestly finite!
- Non-trivial?
- Integrable?
- Conserved?

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If any of these is answered with 'no' then back to square one in algorithm!

Example: Lobachevsky holography

Split boundary preserving gauge trafos into components:

$$\epsilon^{(0)} = \epsilon_1 \, L_1 + \epsilon^+_{\frac{1}{2}} \, \psi^+_{\frac{1}{2}} + \epsilon^-_{\frac{1}{2}} \, \psi^-_{\frac{1}{2}} + \epsilon^L_0 \, L_0 + \epsilon^S_0 \, S + \epsilon^+_{-\frac{1}{2}} \, \psi^+_{-\frac{1}{2}} + \epsilon^-_{-\frac{1}{2}} \, \psi^-_{-\frac{1}{2}} + \epsilon_{-1} \, L_{-1}$$

Solving constraint that gauge trafos generated by $\epsilon^{(0)}$ preserve boundary conditions

$$\partial_\mu \epsilon^{(0)\,a} + f^a{}_{bc} \left(\hat{a}^{(0)}_\mu + a^{(0)}_\mu \right)^b \epsilon^{(0)\,c} = \mathcal{O}(a^{(0)}_\mu)^a$$

yields results for components of $\epsilon^{(0)}$

$$\epsilon_1 = \epsilon(\varphi) \qquad \epsilon^\pm_{\frac{1}{2}} = \epsilon^\pm_{\frac{1}{2}}(\varphi) \qquad \epsilon^L_0 = 4\epsilon'(\varphi) \qquad \epsilon^S_0 = \epsilon_0(\varphi)$$

$$\epsilon^\pm_{-\frac{1}{2}} = 4\epsilon^{\pm\,\prime}_{\frac{1}{2}}(\varphi) \mp \frac{4\pi}{k} \Big(2\mathcal{W}^\pm_{\frac{1}{2}}(\varphi)\epsilon(\varphi) - 3\mathcal{W}_0(\varphi)\epsilon^\pm_{\frac{1}{2}}(\varphi) \Big)$$

$$\epsilon_{-1} = 8\epsilon''(\varphi) + \frac{4\pi}{k} \Big(2\mathcal{L}(\varphi)\epsilon(\varphi) + \mathcal{W}^-_{\frac{1}{2}}(\varphi)\epsilon^+_{\frac{1}{2}}(\varphi) + \mathcal{W}^+_{\frac{1}{2}}(\varphi)\epsilon^-_{\frac{1}{2}}(\varphi) \Big)$$

Canonical charges:

$$Q[\epsilon^{(0)}] = \oint \mathrm{d}\varphi \left(\mathcal{L}\epsilon + \mathcal{W}_0\epsilon_0 + \mathcal{W}^+_{\frac{1}{2}}\epsilon^-_{\frac{1}{2}} + \mathcal{W}^-_{\frac{1}{2}}\epsilon^+_{\frac{1}{2}} \right)$$

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Classical asymptotic symmetry algebra

Dirac bracket algebra of canonical boundary charges:

$$\{Q[\epsilon_1],\, Q[\epsilon_2]\} = \delta_{\epsilon_2}\, Q[\epsilon_1]$$

- Either evaluate left hand side directly (Dirac brackets)
- Or evaluate right hand side (usually easier)

Exactly like in seminal Brown–Henneaux work!

Example: Lobachevsky holography

Dirac bracket algebra of canonical boundary charges:

$$\{\mathcal{L}(\varphi),\mathcal{L}(\bar\varphi)\} = -4\big(2\mathcal{L}\delta'(\varphi-\bar\varphi)-\mathcal{L}'\delta(\varphi-\bar\varphi)\big)-\frac{4k}{\pi}\delta'''(\varphi-\bar\varphi)$$

$$\{\mathcal{L}(\varphi),\mathcal{W}_0(\bar\varphi)\} = 0$$

$$\{\mathcal{L}(\varphi),\mathcal{W}^{\pm}_{\frac{1}{2}}(\bar\varphi)\} = -4\Big(\frac{3}{2}\mathcal{W}^{\pm}_{\frac{1}{2}}\delta'(\varphi-\bar\varphi)-\big(\mathcal{W}^{\pm}_{\frac{1}{2}}{}' \pm \frac{3\pi}{k}\mathcal{W}^{\pm}_{\frac{1}{2}}\mathcal{W}_0\big)\delta(\varphi-\bar\varphi)\Big)$$

$$\{\mathcal{W}_0(\varphi),\mathcal{W}_0(\bar\varphi)\} = \frac{k}{3\pi}\delta'(\varphi-\bar\varphi)$$

$$\{\mathcal{W}_0(\varphi),\mathcal{W}^{\pm}_{\frac{1}{2}}(\bar\varphi)\} = \pm\mathcal{W}^{\pm}_{\frac{1}{2}}\delta(\varphi-\bar\varphi)$$

$$\{\mathcal{W}^{+}_{\frac{1}{2}}(\varphi),\mathcal{W}^{-}_{\frac{1}{2}}(\bar\varphi)\} = \mathcal{L}\delta(\varphi-\bar\varphi)-4\Big(-3\mathcal{W}_0\delta'(\varphi-\bar\varphi)+\big(\frac{3}{2}\mathcal{W}_0{}' - \frac{9\pi}{2k}\mathcal{W}_0\mathcal{W}_0\big)\delta(\varphi-\bar\varphi)-\frac{k}{2\pi}\delta''(\varphi-\bar\varphi)\Big)$$

Note: second and third line require Sugawara-shift

$$\mathcal{L}\to\mathcal{L}-\frac{6\pi}{k}\mathcal{W}_0\mathcal{W}_0\equiv\hat{\mathcal{L}}$$

... continued

Replace Dirac brackets by commutators and make Fourier expansions

$$[J_n, J_m] = -\frac{2k}{3} n\delta_{n+m,0}$$
$$[J_n, \hat{L}_m] = nJ_{n+m}$$
$$[J_n, G^{\pm}_m] = \pm G^{\pm}_{m+n}$$
$$[\hat{L}_n, \hat{L}_m] = (n-m)\hat{L}_{m+n} + \frac{c}{12} n\big(n^2-1\big)\delta_{n+m,0}$$
$$[\hat{L}_n, G^{\pm}_m] = \big(\frac{n}{2} - m\big)G^{\pm}_{n+m}$$
$$[G^+_n, G^-_m] = \hat{L}_{m+n} + \frac{3}{2}(m-n)J_{m+n} + \frac{3}{k}\sum_{p\in\mathbb{Z}} J_{m+n-p}J_p + k\big(n^2 - \frac{1}{4}\big)\delta_{m+n,0}$$

Semi-classical (large k) Polyakov–Bershadsky algebra $W_3^{(2)}$

Note: resembles $N = 2$ superconformal algebra

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Quantum asymptotic symmetry algebra

Introducing normal ordering in expressions like

$$\sum_{p\in\mathbb{Z}} :J_{n-p}J_p: = \sum_{p\geq 0} J_{n-p}J_p + \sum_{p<0} J_p J_{n-p}$$

can make semi-classical algebra inconsistent

First example I am aware of: Henneaux–Rey 2010 in spin-3 AdS gravity

Quantum violations of Jacobi-identities possible!

Resolution: deform suitable structure constants/functions and demand validity of Jacobi identities

Example: Lobachevsky holography

Five deformation parameters in $[J, J]$ and $[G^+, G^-]$
Solving Jacobi identities yields (quantum) Polyakov–Bershadsky algebra

$$[J_n, J_m] = \frac{2\hat{k}+3}{3} n\delta_{n+m,0}$$

$$[J_n, \hat{L}_m] = nJ_{n+m}$$

$$[J_n, \hat{G}^\pm_m] = \pm G^\pm_{m+n}$$

$$[\hat{L}_n, \hat{L}_m] = (n-m)\hat{L}_{m+n} + \frac{\hat{c}}{12} n(n^2-1)\delta_{n+m,0}$$

$$[\hat{L}_n, \hat{G}^\pm_m] = \left(\frac{n}{2} - m\right)\hat{G}^\pm_{n+m}$$

$$[\hat{G}^+_n, \hat{G}^-_m] = -(\hat{k}+3)\hat{L}_{m+n} + \frac{3}{2}(\hat{k}+1)(n-m)J_{m+n} + 3\sum_{p\in\mathbb{Z}} : J_{m+n-p}J_p : \\ + \frac{(\hat{k}+1)(2\hat{k}+3)}{2}(n^2 - \frac{1}{4})\delta_{m+n,0}$$

with central charge $\hat{c} = -(2\hat{k}+3)(3\hat{k}+1)/(\hat{k}+3) = -6\hat{k} + \mathcal{O}(1)$

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Unitary representations of quantum asymptotic symmetry algebra

Standard questions:

- Is $\hat{u}(1)$ level non-negative?
- Is central charge non-negative?
- Are there any negative norm states?
- Are there null states?

To be decided on case-by-case basis!

Example: Lobachevsky holography

Non-negativity of $\hat{u}(1)$ level:

$$\hat{k} \geq -\frac{3}{2}$$

Non-negativity of central charge:

$$-\frac{1}{3} \geq \hat{k} \geq -\frac{3}{2}$$

Norm of vacuum descendants at level $\frac{3}{2}$:

$$K^{(\frac{3}{2})} = (\hat{k}+1)(2\hat{k}+3)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Positive and negative norm states, unless pre-factor vanishes

Only two possible values of level $\hat{k}$ compatible with unitarity:

$$\hat{k} = -1 \qquad \text{or} \qquad \hat{k} = -\frac{3}{2}$$

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Identify or at least constrain dual field theory

Collect all clues and make reasonable guess!

			1			7	4	
	5			9			3	2
		6	7			9		
4			8					
	2						1	
					9			5
		4			7	3		
7	3			2			6	
	6	5			4			

Example: Lobachevsky holography

Unitary case $\hat{k} = -\frac{3}{2}$ has vanishing central charge, $\hat{c} = 0$:

$$\boxed{\text{Only state in theory is vacuum!}}$$

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Unitary case $\hat{k} = -1$:

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- All half-integer states are null states

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Unitary case $\hat{k} = -1$:

- Unity central charge, $\hat{c} = 1$
- All half-integer states are null states
- Virasoro generators determined from current generators:

$$\hat{L}_{-n}|0\rangle = \frac{3}{2}\sum_{p\in\mathbb{Z}} : J_{-p}J_{-n+p} : |0\rangle$$

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- Positive norm states:

$$J_{-n_1}^{m_1} \dots J_{-n_N}^{m_N}|0\rangle$$

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- Unity central charge, $\hat{c} = 1$
- All half-integer states are null states
- Virasoro generators determined from current generators:

$$\hat{L}_{-n}|0\rangle = \frac{3}{2}\sum_{p\in\mathbb{Z}} : J_{-p}J_{-n+p} : |0\rangle$$

- Positive norm states:

$$J_{-n_1}^{m_1} \dots J_{-n_N}^{m_N} |0\rangle$$

- Bar-sector: only affine $\hat{u}(1)$ algebra with positive level

Example: Lobachevsky holography

Unitary case $\hat{k} = -\frac{3}{2}$ has vanishing central charge, $\hat{c} = 0$:

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- Dual CFT: free boson

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Non-AdS holography provides many avenues for future research

Thanks for your attention!

Literature on non-AdS holography in 3D higher spin gravity:

M. Gary, D. Grumiller and R. Rashkov, "Towards non-AdS holography in 3-dimensional higher spin gravity," *JHEP* **1203** (2012) 022, 1201.0013.

H. Afshar, M. Gary, D. Grumiller, R. Rashkov and M. Riegler, "Non-AdS holography in 3-dimensional higher spin gravity," 1209.2860.

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