Instability of extreme black holes

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Extreme black holes

- Extreme black holes do not emit Hawking radiation ($\kappa = 0$). Expect simpler description in quantum gravity.

- This has been realised in string theory: statistical derivation of entropy $S = \frac{A}{4\hbar}$ for certain BPS/supersymmetric black holes.
  [Strominger, Vafa '95]

- Possess a well-defined notion of *near-horizon geometry* which typically have an AdS_2 structure (even non-BPS).
  [Kunduri, JL, Reall '07]

Near-horizon geometry

- Proposal that extreme Kerr black holes can be described by 2d (chiral) CFT. [Guica, Hartman, Song, Strominger '08]

- Near-horizon rigidity: any vacuum axisymmetric near-horizon geometry is given by that of extreme Kerr black hole.
[Hajicek '74; Lewandowski, Pawlowski '02; Kunduri, JL '08]

- Used to extend 4d no-hair theorems to extreme black holes.
[Meinel et al '08; Amsel et al '09; Figueras, JL '09; Chrusciel '10]

Stability of extreme black holes

- Are extreme black holes stable? By this we mean:

> "An initially small perturbation remains small for all time and eventually settles down to a stationary perturbation, which corresponds to a small variation of parameters within the family of black hole solutions which contains the extreme black hole."

Generically this results in a slightly non-extreme black hole. Of course we do not regard this as an instability!

- This talk: (in)stability of extreme black holes in four dimensional General Relativity [Aretakis '11 '12; JL, Reall '12]

But aren't BPS solutions stable?

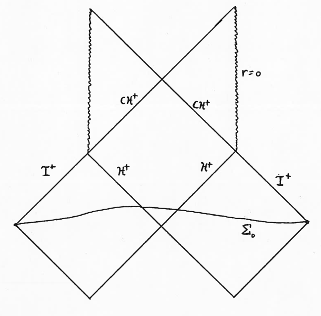
- Extreme black holes often saturate BPS bound $\Longrightarrow$ preserve some supersymmetry. Does this mean they are stable?

- No! Stability of BPS solutions not guaranteed in gravitational theories: no (positive def.) local gravitational energy density...

- Stability of Minkowski space does not follow from positive mass theorem. Required long book [Christodoulou, Klainerman '93]!

Heuristic argument for instability

- Reissner-Nordström black hole
  $\mathcal{H}^+$: event horizon $r = r_+$
  $\mathcal{CH}^+$: inner horizon $r = r_-$
- Infinite blue-shift at $\mathcal{CH}^+ \implies$ inner (Cauchy) horizon unstable and evolves to null singularity.
  [Penrose '68; Israel, Poisson '90]
  [Dafermos '03]



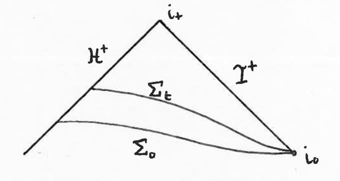
Extreme limit $r_- \to r_+$. Test particles encounter null singularity just as they cross $\mathcal{H}^+$. Expect instability of event horizon of extreme black hole. [Marolf '10]

Stability of non-extreme black holes

- *Mode stability* of linearized *gravitational* perturbations of Schwarzschild and Kerr black holes [Regge, Wheeler '57; Whiting '89].
- Consider simpler toy model. Massless scalar $\nabla^2\psi = 0$ in a *fixed* black hole background, e.g. Schwarzschild.
- Modes $\psi = r^{-1}F(r)Y_{jm}e^{-i\omega t}$ obey $\left[-\frac{d^2}{dr_*^2} + V(r)\right] F = \omega^2 F$. $V \geq 0$ so no unstable modes (i.e. with Im $\omega > 0$).
- This is not enough to establish linear stability! Issues: completeness of mode solutions, infinite superpositions...

Stability of non-extreme black holes

- Prescribe initial data: $\psi, \dot{\psi}$ on Σ_0 which intersects $\mathcal{H}^+$ and infinity with $\psi \to 0$ at infinity.



- **Theorem**: $\psi|_{\Sigma_t} = O(t^{-\alpha})$ for some $\alpha > 0$, as $t \to \infty$, everywhere outside and on $\mathcal{H}^+$. All derivatives of ψ also decay. [Dafermos, Rodnianski '05] (boundedness of ψ by [Kay, Wald '89])

- Similar results shown for *non-extreme* Reissner-Nordström [Blue, Soffer '09] and Kerr [Dafermos, Rodnianski '08 '10]

Redshift effect

- Since ∂_t becomes null on horizon its associated energy density degenerates there. Harder to bound ψ near $\mathcal{H}^+$.
- Stability proofs reveal that *redshift effect* along horizon is important. [Dafermos, Rodnianski '05]
- Redshift factor along $\mathcal{H}^+$ is $\sim e^{-\kappa v}$ where κ is surface gravity. For $\kappa > 0$ this leads to redshift effect.
- For extreme black holes $\kappa = 0$ so no redshift effect...

Scalar instability of extreme black holes

- Aretakis has shown that a massless scalar ψ in extreme Reissner-Norström is *unstable* at horizon. [Aretakis '11]

- He proved that ψ decays on and outside $\mathcal{H}^+$. However, derivatives transverse to the horizon do not decay!

- Advanced time and radial coords (v, r). For generic initial data, as $v \to \infty$, $\partial_r \psi|_{\mathcal{H}^+}$ does not decay and $\partial_r^k \psi|_{\mathcal{H}^+} \sim v^{k-1}$.

- Analogous results for extreme Kerr. [Aretakis '11 '12]

Conservation law along horizon

- Write RN in coordinates regular on future horizon $\mathcal{H}^+$:

$$ds^2 = -F(r)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}r + r^2\mathrm{d}\Omega^2$$

Horizon at $r = r_+$, largest root of F. Extreme iff $F'(r_+) = 0$.

- Evaluate wave equation on $\mathcal{H}^+$, i.e. at $r = r_+$:

$$\nabla^2\psi|_{\mathcal{H}^+} = 2\frac{\partial}{\partial v}\left(\partial_r\psi + \frac{1}{r_+}\psi\right) + F'(r_+)\partial_r\psi + \hat{\nabla}^2_{S^2}\psi$$

- Extreme case: for spherically symmetric ψ_0,

$$I_0[\psi] \equiv \partial_r\psi_0 + \frac{1}{r_+}\psi_0$$

is independent of v, i.e. conserved along $\mathcal{H}^+$!

Blow up along horizon

- Generic initial data $I_0 \neq 0$. Hence $\partial_r \psi_0$ and ψ_0 cannot both decay along $\mathcal{H}^+$! Actually ψ_0 decays: hence $\partial_r \psi_0 \to I_0$!
- Now take a transverse derivative of $\nabla^2 \psi$ and evaluate on $\mathcal{H}^+$:

$$\partial_r (\nabla^2 \psi)|_{\mathcal{H}^+} = \frac{\partial}{\partial v} \left(\partial_r^2 \psi + \frac{1}{r_+} \partial_r \psi \right) + \frac{2}{r_+^2} \partial_r \psi$$

- Hence as $v \to \infty$ we have $\partial_v (\partial_r^2 \psi_0) \to -2I_0/r_+^2$ and therefore

$$\partial_r^2 \psi_0 \sim - \left(\frac{2I_0}{r_+^2} \right) v$$

blows up along $\mathcal{H}^+$. Instability!

Higher order quantities

- Higher derivatives blow up faster $\partial_r^k \psi_0 \sim c I_0 v^{k-1}$.
- Let ψ_j be projection of ψ onto Y_{jm}. Then for any solution to $\nabla^2 \psi = 0$ one has a *hierarchy of conserved quantities*

$$I_j[\psi] = \partial_r^{j+1} \psi_j + \sum_{i=1}^{j} \beta_i \partial_r^i \psi_j$$

and $\partial_r^{j+k} \psi_j \sim c I_j v^{k-1}$ as $v \to \infty$.

- Analogous tower of conservation laws and blow up for *axisymmetric* perturbations of extreme Kerr.

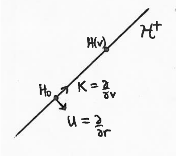
General extreme horizons

- Aretakis's argument can be generalised to cover all known D-dimensional extreme black holes. [JL, Reall '12]

- Consider a degenerate horizon $\mathcal{H}^+$ with a *compact* spatial section H_0 with coords x^a. Gaussian null coordinates:

$$ds^2 = 2\,dv\,(dr + r\,h_a dx^a + \tfrac{1}{2} r^2 F dv) + \gamma_{ab} dx^a dx^b$$

where $\mathcal{H}^+$ is at $r = 0$ and $K = \partial/\partial v$ is Killing vector.



Conserved quantity

- Change parameter $r \to \Gamma(x) r$ for $\Gamma(x) > 0$. Then metric has same form with $h \to \Gamma h + d\Gamma$. Use this to fix $\nabla_a h^a = 0$.
- $\Gamma(x)$ corresponds to a preferred affine parameter r for the geodesics U. (Appears in AdS_2 of near-horizon geometry).
- Evaluate $\nabla^2 \psi = 0$ on $\mathcal{H}^+$ and assume $H(v)$ compact. Then

$$I_0 = \int_{H(v)} \sqrt{\gamma} \left(2\partial_r \psi + \partial_r (\log \sqrt{\gamma}) \psi \right)$$

is independent of v, i.e. it is conserved along $\mathcal{H}^+$.

Scalar instability for general extreme horizons

- Let $A \equiv (F - h^a h_a)/\Gamma$. Evaluating $\partial_r(\nabla^2\psi)$ on $\mathcal{H}^+$ gives

$$\partial_v J(v) = 2\int_{H(v)} \sqrt{\gamma}[A\partial_r\psi + B\psi]$$

  where $J(v) \equiv \int_{H(v)} \sqrt{\gamma}\left[\partial_r^2\psi + \ldots\right]$.

- Suppose $\psi \to 0$ as $v \to \infty$. If $A = A_0 \neq 0$ is constant and $I_0 \neq 0$ then $\partial_v J \to A_0 I_0$ and $J(v) \sim A_0 I_0 v$ blows up.

- A determined by *near-horizon geometry*: negative constant for all known extreme black holes due to AdS_2-symmetry. [JL '12]

Gravitational perturbations

- Study of solutions to *linearized* Einstein equations much more complicated. Issues: gauge, decoupling, (separability)...
- Remarkable fact. Spin-s perturbations of Kerr decouple: $\mathcal{T}_s(\Psi_s) = 0$ for single gauge invariant complex scalar Ψ_s.
  [Teukosky '74]
- Gravitational variables $s = \pm 2$: null tetrad $(\ell, n, m, \bar{m})$ and Ψ_s is a Weyl scalar $\delta\psi_0$ ($s = 2$) or $\delta\psi_4$ ($s = -2$) where:

$$\psi_0 = C_{\mu\nu\rho\sigma}\ell^\mu m^\nu \ell^\rho \bar{m}^\sigma \qquad \psi_4 = C_{\mu\nu\rho\sigma} n^\mu m^\nu n^\rho \bar{m}^\sigma$$

Gravitational perturbations of Kerr

- It is believed that *non-extreme* Kerr black hole is stable: evidence from massless scalar, mode stability, simulations...

- Aretakis's scalar instability for extreme Kerr be generalised to higher spin fields! Electromagnetic $s = \pm 1$ and most importantly *gravitational* perturbations $s = \pm 2$. [JL, Reall '12]

- Use tetrad and coords (v, r, θ, ϕ) which are *regular* on $\mathcal{H}^+$. Horizon at largest root r_+ of $\Delta(r) = r^2 - 2mr + a^2$.

Gravitational perturbations of Kerr

- Teukolsky equation for a spin s-field ψ takes simple form:

$$\frac{\partial}{\partial v}\left\{N(\psi) + 2a\frac{\partial\psi}{\partial\phi} + 2[(1-2s)r - ias\cos\theta]\psi\right\}$$
$$= \mathcal{O}_s\psi - \Delta\frac{\partial^2\psi}{\partial r^2} - (1-s)\Delta'\frac{\partial\psi}{\partial r} - 2a\frac{\partial^2\psi}{\partial\phi\partial r}$$

  $N = 2(r^2+a^2)\frac{\partial}{\partial r} + a^2\sin^2\theta\frac{\partial}{\partial v}$ is a transverse vector to $\mathcal{H}^+$ ($N^\mu \sim U^\mu$ on horizon).

- Operator $\mathcal{O}_s$ is diagonalised by the spin weighted spheroidal harmonics ${}_sY_{jm}(\theta,\phi)$ where $j \geq |s|$ and $j \geq |m|$.

  Non-trivial kernel iff $s \leq 0$ with $j = -s$.

Teukolsky equation for extreme Kerr

- Restrict to extreme Kerr $\Delta'(r_+) = 0$. Evaluate Teukolsky for $s \leq 0$ at $r = r_+$ and project to axisymmetric scalar $\mathcal{O}_s \psi = 0$ (i.e. $j = -s, m = 0$).

- RHS of Teukolsky vanish giving 1st-order conserved quantity

$$I_0^{(s)} = \int_{H(v)} d\Omega \, ({}_sY_{-s0})^* [N(\psi) + f(\theta)\psi]$$

- $I_0^{(s)} \neq 0$ for generic initial data on Σ_0. Hence ψ and the $j = -s$ component of $N(\psi)$ cannot both decay!

Teukolsky equation for extreme Kerr

- Take transverse derivative $N|_{r=r_+}$ of Teukolsky equation:

$$\partial_v J^s(v) = -2(1-s)\int_{H(v)} \mathrm{d}\Omega \; ({}_sY_{-s0})^* N(\psi)$$

where $J^s = \int_{H(v)} \mathrm{d}\Omega \; ({}_sY_{-s0})^*[N^2(\psi) + f(\theta)N(\psi) + g(\theta)\psi]$.

- If $\psi \to 0$ as $v \to \infty$ this shows $J^s(v) \sim -2(1-s)I_0^{(s)} v$ $\implies N^2(\psi)_{j=-s}$ or $N(\psi)$ must blow up at least linearly in v.

- Can derive $p+1$ order conserved quantities $I_p^{(s)}$ by applying N p-times to Teukolsky equation. For $s > 0$ turns out $p \geq 2s$.

Gravitational instability of extreme Kerr

- If extreme Kerr is stable then for any perturbation, at large v it must approach a nearby member of the Kerr family.
- The Kerr solution has $\psi_0 = \psi_4 \equiv 0$ (type D). Hence if stable, perturbations $\delta\psi_0, \delta\psi_4 \to 0$ for large $v \implies I_p^{(s)} = 0$.

  Any axisymmetric initial data with $I_p^{(s)} \neq 0$ leads to instability!

Summary of most well-behaved possibility

Along $\mathcal{H}^+$: $\delta\psi_4$ decays, $N(\delta\psi_4)$ does not, $N^2(\delta\psi_4)$ blows up; $N^{0\leq k\leq 4}(\delta\psi_0)$ all decay, $N^5(\delta\psi_0)$ does not, $N^6(\delta\psi_0)$ blows up.

Most tangential comps of Weyl ($\delta\psi_4$) exhibit worst behaviour.

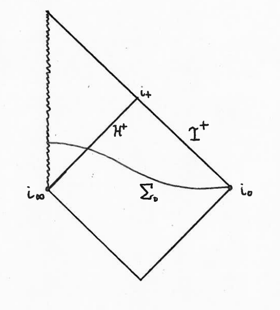
Possible endpoints of instability?

- Need full non-linear evolution to answer this properly. Some possibilities:

  1. An initially small perturbation becomes large, but still eventually settles down to a near extreme Kerr.
  2. Horizon becomes a null singularity. [Marolf '10]
  3. Something else?

Choice of initial data

Initial data surface Σ_0 is not complete for extreme black holes; any surface crossing $\mathcal{H}^+$ must intersect the singularity. ($\exists$ complete surfaces which end in AdS_2 throat i_∞, but then need asymptotic conditions...)



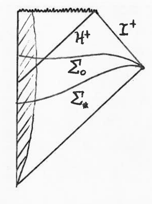
So how do we know what perturbations are actually allowed? We assumed that for generic initial data the various conserved quantities are non-zero. Is this really true?

Choice of initial data

- Extreme RN can be formed by gravitational collapse (e.g. spherical shell of charged matter [Kuchar '68; Farrugia, Hajicek '79]).

Now there exists *complete* Σ_0 intersecting matter fallen behind $\mathcal{H}^+$.

Data on Σ_0 defined from unique Cauchy evolution of data on complete surface Σ_* which coresponds to before black hole forms.



- Arbitrary smooth data on $\Sigma_* \implies$ arbitrary data on Σ_0, so generically $I_0 \neq 0$. Expect same for Kerr.

Summary

- **Main results**

  Linearized gravitational instability of extreme Kerr black hole!

  Instability of massless scalar on horizon of any extreme black hole. Transverse derivatives blow up along horizon.

  These results are in marked contrast to the non-extreme case.

- **Open problems**

  Physical interpretation of conservation laws along horizon.

  Generalizations to higher dimensional extreme black objects.

  Implication of instabilities within GR and string theory.