Near-Extremal Vanishing Horizon AdS_5 Black Holes and Their CFT Duals

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Introduction

Black Holes

1. solutions to general relativity
2. behave like thermodynamic systems:
    - satisfy thermodynamic laws
    - have a thermodynamic entropy:

$$S_{BH} = \frac{A_d}{4G_d}$$

Question

- Why does the entropy scale like the horizon area? $\Rightarrow$ Holography: "the fundamental degrees of freedom describing the system are described by a quantum field theory with one less dimension."

Introduction

Question

- What are the the underlying states of this QFT giving rise to black hole entropy?
- Two commonly used tools:

1. Near horizon geometry: Zoom in on region very close to the event horizon r_+.
2. Extremality: T=0 black holes are more symmetric: AdS_2 factor in near horizon geometry

Introduction

Kerr/CFT (Extremal Black Hole/CFT) Correspondence

- Statement of Kerr/CFT:
  Near horizon quantum states $\Longleftrightarrow$ quantum states of a chiral 2d CFT

Introduction

Chiral 2d CFT

- 2d CFT: 2d quantum field theory invariant under conformal transformations. Generators L_n of conformal transformations obey Virasoro algebra:

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}.$$

- Central charge c: a number that characterises the CFT
- States in 2d CFT: split into left-moving and right-moving pieces in left and right moving sectors.
- Left-moving sector: L_m, L_n; c_L. Right-moving sector: $\bar{L}_m, \bar{L}_n$; c_R.
- Chiral 2d CFT: excited states exist in only the left-moving sector. One copy of Virasoro algebra with one c_L.

Introduction

Kerr/CFT (Extremal Black Hole/CFT) Correspondence

- Statement of Kerr/CFT: Extremal black holes are holographically dual to chiral 2d conformal field theory.
- Near horizon geometry: $ds^2 = ds^2_{AdS_2} + \ldots$
- Use near horizon data to compute
  1. c_L
  2. Frolov-Thorne temperature T_L: "temperature of the dual CFT".
- Microscopic Cardy formula $\Rightarrow$ macroscopic black hole entropy:

$$\boxed{S_{Cardy} = \frac{\pi^2}{3} c_L T_L = S_{BH}}$$

Introduction

Kerr/CFT (Extremal Black Hole/CFT) Correspondence

- Kerr/CFT: originally for 4d black holes. Generalised to higher dimensions.
- Vacuum degeneracy of chiral 2d CFT accounts for macroscopic black hole entropy.
- Little more information.

Introduction

AdS/CFT Correspondence

- AdS/CFT Correspondence: Gravity in $AdS_{d+1} \Longleftrightarrow CFT_d$.
- 1:1 correspondence between local fields in the gravity theory and operators in the boundary QFT.
- AdS_3/CFT_2: non-chiral 2d CFT dual to gravity in AdS_3.

Question

Can an extremal black hole have a near horizon AdS_3 throat that's dual to the full non-chiral CFT_2?

Introduction

Q: Can an Extremal Black Hole have a Near Horizon AdS_3?

- Answer: Yes
- A_H, $T_H \to 0$: Extremal Vanishing Horizon (EVH) black holes.
- EVH black holes: Near horizon geometry develops locally AdS_3 throat.
- Local AdS_3 near horizon $\Rightarrow$ dual CFT_2 description: EVH/CFT Correspondence.
- A_H, $T_H \sim \epsilon << 1$: Near-EVH black holes: $AdS_3 \to$ BTZ black hole.
- Asymptotically $AdS_5 \times S^5$ (near-)EVH black holes: 4d CFT dual: link with near horizon 2d CFT?

Plan of the Talk

1. Describe asymptotically $AdS_5 \times S^5$ black hole solutions to 10d IIB supergravity
2. Criteria: EVH and near-EVH black holes
3. Near horizon limit: AdS_3
4. IR dual CFT_2 and compare with UV CFT_4
5. 1st Law of Thermodynamics in near-EVH limit
6. Compare results with Kerr/CFT
7. Summarise and Discuss

5d Supergravity Solution

- **Black hole solution** to U(1)^3 5d gauged supergravity:

$$\begin{aligned} ds^2 &= H^{-\frac{4}{3}}\Big[-\frac{X}{\rho^2}(dt - a\sin^2\theta\,\frac{d\phi}{\Xi_a} - b\cos^2\theta\,\frac{d\psi}{\Xi_b})^2 \\ &\quad +\frac{C}{\rho^2}(\frac{ab}{f_3}dt - \frac{b}{f_2}\sin^2\theta\frac{d\phi}{\Xi_a} - \frac{a}{f_1}\cos^2\theta\frac{d\psi}{\Xi_b})^2 \\ &+ \frac{Z\sin^2\theta}{\rho^2}(\frac{a}{f_3}dt - \frac{1}{f_2}\frac{d\phi}{\Xi_a})^2 + \frac{W\cos^2\theta}{\rho^2}(\frac{b}{f_3}dt - \frac{1}{f_1}\frac{d\psi}{\Xi_b})^2\Big] \\ &\quad +H^{\frac{2}{3}}(\frac{\rho^2}{X}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2)\,, \end{aligned}$$

- Gauge fields:

$$\begin{aligned} A^1 &= A^2 = P_1(dt - a\sin^2\theta\,\frac{d\phi}{\Xi_a} - b\cos^2\theta\,\frac{d\psi}{\Xi_b}) \\ A^3 &= P_3(b\sin^2\theta\,\frac{d\phi}{\Xi_a} + a\cos^2\theta\,\frac{d\psi}{\Xi_b}) \end{aligned}$$

- Scalar fields:

$$X_1 = X_2 = H^{-\frac{1}{3}}, \quad X_3 = H^{\frac{2}{3}}$$

- $H, \rho, \tilde{\rho}, f_i, \Delta_\theta, C, Z, W, \Xi_a, \Xi_b, P_i$: functions of $(r; a, b, q, m)$.
- **Horizon function:** $X(r_+) = X(r_-) = 0$

$$\boxed{X(r) = \frac{1}{r^2}(a^2 + r^2)(b^2 + r^2) - 2m + \frac{(a^2 + r^2 + q)(b^2 + r^2 + q)}{\ell^2}}$$

Thermodynamic Quantities

- **Hawking Temperature**:

$$T_{\mathrm{H}} = \frac{2r_+^6 + r_+^4(\ell^2 + a^2 + b^2 + 2q) - a^2 b^2 \ell^2}{2\pi r_+ \ell^2[(r_+^2 + a^2)(r_+^2 + b^2) + q r_+^2]}$$

- **Beckenstein-Hawking Entropy:**

$$S_{\mathrm{BH}} = \frac{\pi^2[(r_+^2 + a^2)(r_+^2 + b^2) + q r_+^2]}{2G_5 \Xi_a \Xi_b r_+}$$

Thermodynamic Quantities

- **Rotation in ϕ, ψ:**
- Angular velocities:

$$\Omega_a = \frac{a(r_+^4 + r_+^2 b^2 + r_+^2 q + \ell^2 b^2 + \ell^2 r_+^2)}{\ell^2(a^2 + r_+^2)(b^2 + r_+^2) + \ell^2 q r_+^2},$$

$$\Omega_b = \frac{b(r_+^4 + r_+^2 a^2 + r_+^2 q + \ell^2 a^2 + \ell^2 r_+^2)}{\ell^2(a^2 + r_+^2)(b^2 + r_+^2) + \ell^2 q r_+^2}.$$

- Angular momenta:

$$J_a = \frac{\pi a\,(2m + q\,\Xi_b)}{4G_5 \Xi_b\,\Xi_a^2}, \qquad J_b = \frac{\pi b\,(2m + q\,\Xi_a)}{4G_5 \Xi_a\,\Xi_b^2}.$$

- parametrised by a,b.

Thermodynamic Quantities

- **Gauge Fields A_i:**
- Chemical Potentials:

$$\Phi_1 = \Phi_2 = \frac{\sqrt{q^2 + 2mq}\; r_+^2}{(a^2 + r_+^2)(b^2 + r_+^2)\; + q r_+^2},$$
$$\Phi_3 = \frac{qab}{(a^2 + r_+^2)(b^2 + r_+^2)\; + q r_+^2}.$$

- Electric Charges:

$$Q_1 = \quad Q_2 = \frac{\pi\sqrt{q^2 + 2mq}}{4G_5 \Xi_a \Xi_b}, \quad Q_3 = -\frac{\pi abq}{4G_5 \ell^2 \Xi_a \Xi_b}.$$

- parametrised by q.
- **Note:** $Q_3 \sim$ ab not independent.

Thermodynamic Quantities

- **First Law of Thermodynamics:**

$$T_{\rm H}\, dS_{\rm BH} = dE - \Omega_a\, dJ_a - \Omega_b\, dJ_b - \sum_{i=1}^{3} \Phi_i\, dQ_i$$

- Integrate $\Rightarrow$ **Black hole mass:**

$$E = \frac{\pi[2m(2\Xi_a + 2\Xi_b - \Xi_a\Xi_b) + q(2\Xi_a^2 + 2\Xi_b^2 + 2\Xi_a\Xi_b - \Xi_a^2\Xi_b - \Xi_b^2}{8G_5\Xi_a^2\Xi_b^2}$$

10d Embedding

- Solution to 10d IIB supergravity:

$$\boxed{ds_{10}^2 = \sqrt{\tilde{\Delta}}\, ds_5^2 + \frac{\ell^2}{\sqrt{\tilde{\Delta}}} d\sum_5^2}$$

- ds_5^2: 5d black hole metric
- deformed S^5:

$$d\sum_5^2 = \sum_{i=1}^{3} X_i^{-1}\left(d\mu_i^2 + \mu_i^2 (d\psi_i + A^i/\ell)^2\right).$$

- also: $F_5 = \star F_5$ with flux N
- Newton's constants:

$$\boxed{G_5 = G_{10}\frac{1}{\pi^3 \ell^5} = \frac{\pi}{2}\frac{\ell^3}{N^2}}.$$

10d Embedding

10d Embedding

- 5d electrostatic potential Φ_i = 10d angular velocity Ω_i on S^5.
- 5d electric charge Q_i = 10d angular momentum J_i on S^5.

Dual 4d Description

- AdS/CFT:

Black Hole in $AdS_5 \times S^5 \leftrightarrow$ mixed state in dual $\mathcal{N} = 4$ SYM.

- **States carry conserved charges given by gravity conserved charges:**

$$\Delta = \ell E\,, \qquad \mathcal{J}_1 = \mathcal{J}_2 = Q_1 = \frac{\sqrt{q^2 + 2mq}}{2\ell^2 \Xi_a \Xi_b} N^2\,,$$

$$\mathcal{S}_a = J_a = \frac{a(2m + q\Xi_b)}{2\ell^3 \Xi_a^2 \Xi_b} N^2\,, \qquad \mathcal{S}_b = J_b = \frac{b(2m + q\Xi_a)}{2\ell^3 \Xi_b^2 \Xi_a} N^2\,.$$

The Set of EVH Black Holes

EVH Black Holes

- Horizon equation: $X(r_+) = 0 \Rightarrow m = m(r_+)$
- 4-dimensional black hole parameter space: $(a, b, q, m) \leftrightarrow (a, b, q, r_+)$
- EVH black holes: $A_{BH} = T_H = 0 \Rightarrow$

$$\boxed{r_+ = 0 \quad \text{and} \quad ab = 0\,.}$$

- Two types of EVH configurations for these black holes:
  1. Rotating: $b = r_+ = 0, \quad a \neq 0$ ($J_b = 0$, $J_a \neq 0$)
  2. Static: $a = b = r_+ = 0$ ($J_a = J_b = 0$)
- Note: EVH limit $\Rightarrow$ angular momentum $\sim$ ab:$J_3 = 0$

The Set of EVH Black Holes

Each EVH Configuration Defines a Surface in Parameter Space

1. Rotating: $X(r_+) = 0 = b$ gives
$$m = \frac{q^2 + a^2(\ell^2 + q)}{2\ell^2} .$$
2. Static: $X(r_+) = 0 = a = b$ gives
$$m = \frac{q^2}{2\ell^2} .$$

Point on the EVH surface $\Leftrightarrow$ EVH black hole

The Near Horizon Limit of EVH Black Holes: Rotating Case $r_+ = b = 0$

- Rotating EVH black hole: $S = T = r_+ = b = 0$:
- Define angles χ, ξ: linear combinations of angles corresponding to vanishing charges:

$$\boxed{\chi = \omega_1 \psi + \omega_2 \psi_3, \qquad \xi = \omega_3 \left(\psi + \frac{al}{q} \psi_3 \right)}$$

where $\omega_1 = \omega_1(\omega_2, \omega_3)$.

- Near Horizon Limit:

$$t = \frac{K}{\epsilon} \tau\,, \qquad \chi = \frac{\tilde{\chi}}{\epsilon}, \qquad r = \epsilon \frac{x}{K}\,, \quad \left(K = \sqrt{\frac{\ell^2 (a^2 + q)}{a^2 \ell^2 + q^2}} \right)$$

and some angular shifts.

The Near Horizon Limit of EVH Black Holes: Rotating Case $r_+ = b = 0$

- Take $\epsilon \to 0$:
- **Near Horizon Geometry:** $\boxed{ds^2 = h_1 h_2 ds^2_{AdS_3} + ds^2_{M_7}}$,

where

$$\boxed{ds^2_{AdS_3} = -\frac{x^2}{\ell_3^2} d\tau^2 + \frac{\ell_3^2 dx^2}{x^2} + x^2 d\tilde{\chi}^2}, \text{ and}$$

$$ds^2_{M_7} = \frac{(a^2+q)h_1 h_2}{\Delta_\theta} d\theta^2 + \frac{\ell^2 \cos^2\alpha \cos^2\theta}{K^2 h_1 h_2} d\xi^2 + \frac{a^2+q}{\Xi_a^2}\frac{h_2}{h_1^3}\Delta_\theta \sin^2\theta d\tilde{\phi}^2 + \ell^2 \frac{h_2}{h_1} d\alpha^2 + \ell^2 \frac{h_1}{h_2}\sin^2\alpha d\beta^2 + \ell^2 \frac{h_1}{h_2}\left[\sum_{i=1,2} \mu_i^2 (d\tilde{\psi}_i - A d\tilde{\phi})^2\right].$$

The Near Horizon Limit of EVH Black Holes: Rotating Case $r_+ = b = 0$

- **Near Horizon Geometry:** $\boxed{ds^2 = h_1 h_2 ds^2_{AdS_3} + ds^2_{M_7}}$, where

$$\boxed{ds^2_{AdS_3} = -\frac{x^2}{\ell_3^2} d\tau^2 + \frac{\ell_3^2 dx^2}{x^2} + x^2 d\tilde{\chi}^2}, \text{ and}$$

- warping factor: $h_1^2 = \frac{a^2\cos^2\theta + q}{a^2+q}$, $h_2^2 = \frac{a^2\cos^2\theta + q\mu_3^2}{a^2+q}$.
- Locally $AdS_3 \times M_7$.
- AdS_3 radius is function of EVH parameters:

$$\ell_3^2 = \frac{a^2+q}{\mathbf{V}} = \frac{a^2+q}{1+\frac{2q}{\ell^2}+\frac{a^2}{\ell^2}}.$$

The Near Horizon Limit of EVH Black Holes: Rotating Case $r_+ = b = 0$

AdS_3 Circle:

- AdS_3 circle $\hat{\chi}$:
- $\chi = \frac{\hat{\chi}}{\epsilon} \Rightarrow \hat{\chi} = \hat{\chi} + 2\pi\epsilon$: Vanishing Periodicity. Locally AdS_3 structure is a **pinching AdS_3**.

The Near Horizon Limit of EVH Black Holes: Static Case
$r_+ = a = b = 0$

- Static EVH black hole: $r_+ = a = b = 0$
- Static EVH Near Horizon Limit:

$$t = \frac{\ell}{\sqrt{q}}\frac{\tau}{\epsilon}, \quad \psi_3 = -\frac{\tilde{\chi}}{\epsilon}, \quad r = \epsilon\frac{\sqrt{q}}{\ell}x\,,$$

and some angular shifts.

- Take $\epsilon \to 0$:
- **Near horizon geometry**: $ds^2 = \mu_3 ds_6^2 + ds_{M_4}^2$ where

$$ds_6^2 = -\frac{x^2 d\tau^2}{\ell_3^2} + \frac{\ell_3^2 dx^2}{x^2} + x^2 d\tilde{\chi}^2 + q(d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2)$$

and
$$ds_{M_4}^2 = \frac{\ell^2}{\mu_3} \sum_{i=1,2} (d\mu_i^2 + \mu_i^2 d\tilde{\psi}_i^2)\,.$$

- **Near horizon geometry**: warped locally $\mathbf{AdS}_3\times\mathbf{S}^3$

$$ds_6^2 = -\frac{x^2 d\tau^2}{\ell_3^2} + \frac{\ell_3^2 dx^2}{x^2} + x^2 d\tilde{\chi}^2 + q(d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2)$$

- AdS_3 and S^3 radii are functions of EVH point:

$$R^2_{AdS_3} = \ell_3^2 = \frac{q}{\mathbf{V}_s}\,, \qquad R^2_{S^3} = q\,.$$

- $2\pi\epsilon$ periodicity in $\tilde{\chi}$: the local AdS_3 throat is the *pinching* AdS_3 orbifold.

EVH Black Hole

- EVH Black Hole: Point on EVH surface
- Near Horizon Geometry: pinching AdS_3

Given a generic EVH point, one can decompose the space of deformations into *tangential* and *orthogonal*.

- Tangential deformations: take us from one EVH black hole to a different one on the EVH hyperplane.
- Orthogonal deformations: excitations of an EVH black hole $\Rightarrow$ *near-EVH black holes*.
- **Near-EVH black holes** $A_{\rm BH}, T_{\rm H} \sim \epsilon \rightarrow 0 \Rightarrow$:

$$\boxed{A_{\rm BH} \sim T_{\rm H} \sim \epsilon \Rightarrow r_+ \sim \epsilon\,, \quad ab \sim \epsilon^2}$$

1. Rotating: $b \sim \epsilon^2, a \sim 1$
2. Static: $a \sim b \sim \epsilon$

Near-EVH Rotating Black Holes

- Rotating near-EVH configuration:

$$\boxed{b : 0 \to \epsilon^2 \hat{b}; \qquad m : m \to m + \epsilon^2 M}$$

- physical excitations of rotating EVH black holes are described by deformation parameters $(M, \hat{b})$.
- The horizon is non-zero in this case; from the horizon equation we have $r_\pm^2 = \epsilon^2 x_\pm^2$ where

$$x_\pm^2 = K^2 \frac{r_\pm^2}{\epsilon^2} = \frac{\ell^2(a^2+q)}{q^2+a^2\ell^2}\left[\frac{\mathbf{W}M \pm \sqrt{\mathbf{W}^2 M^2 - \mathbf{V} a^2 \hat{b}^2}}{\mathbf{V}}\right]$$

Near Horizon Geometry: Rotating near-EVH Case

- Near Horizon Limit: same as for EVH case.
- **Near horizon geometry:** $\boxed{ds^2 = h_1 h_2 ds^2_{BTZ} + ds^2_{M_7}}$,
  where $ds^2_{M_7}$ is as for the EVH case, and

$$ds^2_{BTZ} = -\frac{(x^2 - x_+^2)(x^2 - x_-^2)}{\ell_3^2 x^2} d\tau^2 + \frac{\ell_3^2 x^2 dx^2}{(x^2 - x_+^2)(x^2 - x_-^2)} + x^2 \left(d\tilde{\chi} - \frac{x_+ x_-}{\ell_3 x^2} d\tau \right)^2$$

- $\hat{\chi} \sim \hat{\chi} + 2\pi\epsilon$: **pinching BTZ black hole.**
- $x_\pm = x_\pm(\hat{b}, M)$.
- Near-EVH limit: **NH pinching AdS_3 excited to NH pinching BTZ**

Near Horizon Geometry: Rotating near-EVH case

- BTZ thermodynamic quantities: need G_3.
- Compactify 10d type IIB supergravity action to 3d:

$$\frac{1}{16\pi G_{10}}\int\sqrt{-g_{(10)}}\left({}^{10}\mathcal{R}+\cdots\right)=\frac{1}{16\pi G_3}\int\sqrt{-g_{(3)}}\left({}^{3}\mathcal{R}+\cdots\right)$$

- 3d Newton's constant:

$$\frac{1}{G_3}=\frac{2N^2\sqrt{(a^2\ell^2+q^2)(a^2+q)}}{\Xi_a\ell^4}\,.$$

Near Horizon Geometry: Rotating near-EVH case

- **BTZ temperature agrees with the 10d temperature up to NH scaling**:

$$T_{\rm BTZ} \equiv \frac{x_+^2 - x_-^2}{2\pi x_+ \ell_3^2} = \frac{K}{\epsilon} T_{\rm H}\,.$$

- **BTZ Entropy, Mass, Angular Momentum inluding pinching:**

$$S_{\rm BTZ} \equiv \frac{2\pi\epsilon \cdot x_+}{4G_3} = S_{\rm BH}\,,$$

$$\ell_3 M_{\rm BTZ} = \frac{x_+^2 + x_-^2}{8\ell_3 G_3}\epsilon = \frac{\ell_3 K}{2\ell^3 \Xi_a}\, M\mathbf{W}\; N^2 \epsilon\,,$$

$$J_{\rm BTZ} = \frac{x_+ x_-}{4\ell_3 G_3}\epsilon = \frac{\ell_3 K}{2\ell^3 \Xi_a}\, a\hat{b}\sqrt{\mathbf{V}}\; N^2 \epsilon\,.$$

Near-EVH Static Black Holes

- Static near-EVH configuration:

$$\boxed{a : 0 \to \epsilon\hat{a}; \qquad b : 0 \to \epsilon\hat{b}; \qquad m : m \to m + \epsilon^2 M}$$

- physical excitations of static EVH black holes described by deformation parameters $(M, \hat{a}, \hat{b})$.
- The horizon is non-zero in this case; from the horizon equation we have $r_\pm^2 = \epsilon^2 x_\pm^2$ where

$$x_\pm^2 = \frac{\ell^2}{2q\mathbf{V}_s}\left(2\mathbf{W}_s M - \mathbf{Y}_s(\hat{a}^2 + \hat{b}^2) \pm \sqrt{\left(2\mathbf{W}_s M - \mathbf{Y}_s(\hat{a}^2 + \hat{b}^2)\right)^2 - 4\mathbf{V}$$

Near Horizon Geometry: Static near-EVH Case

- Near Horizon Limit: same as for EVH case.
- **Near horizon geometry:** $\boxed{ds^2 = \mu_3 ds_6^2 + ds_{M_4}^2}$, where

$ds_{M_4}^2$ is as for the EVH case, and

$$ds_6^2 = -\frac{(x^2 - x_+^2)(x^2 - x_-^2)}{\ell_3^2 x^2} d\tau^2 + \frac{\ell_3^2 x^2 dx^2}{(x^2 - x_+^2)(x^2 - x_-^2)} + x^2 (d\tilde{\psi}_3 - \frac{x_+ x_-}{\ell_3 x^2} d\tau)^2 \Big] + q(d\theta^2 + \sin^2\theta d(\phi - \frac{\hat{a}}{q}\frac{\ell}{\sqrt{q}}\tau - \frac{\hat{b}\ell}{q}\tilde{\chi})^2 + \cos^2\theta d(\psi - \frac{\hat{b}}{q}\frac{\ell}{\sqrt{q}}\tau - \frac{\hat{a}\ell}{q}\tilde{\chi})^2)$$

- **local BTZ black hole non-trivially fibred by rotating S^3.**
- $\hat{\chi} \sim \hat{\chi} + 2\pi\epsilon$: **pinching BTZ black hole.**
- $x_\pm = x_\pm(\hat{a}, \hat{b}, M)$.

Near Horizon Geometry: Static near-EVH case

- BTZ thermodynamic quantities: need G_3.
- Compactify 10d type IIB supergravity action to 3d:

$$\frac{1}{16\pi G_{10}}\int\sqrt{-g_{(10)}}\left({}^{10}\mathcal{R}+\cdots\right)=\frac{1}{16\pi G_3}\int\sqrt{-g_{(3)}}\left({}^{3}\mathcal{R}+\cdots\right)$$

- 3d Newton's constant:

$$\frac{1}{G_3}=\frac{q^{3/2}\ell^4}{16G_{10}}(2\pi)^4=\frac{2q^{\frac{3}{2}}N^2}{\ell^4}. \tag{1}$$

Near Horizon Geometry: Static near-EVH case

- **BTZ temperature agrees with the 10d temperature up to NH scaling**:

$$T_{\rm BTZ} \equiv \frac{x_+^2 - x_-^2}{2\pi x_+ \ell_3^2} = \frac{\ell}{\epsilon\sqrt{q}}\ T_{\rm H}$$

- **BTZ Entropy, Mass, Angular Momentum:**

$$S_{\rm BTZ} \equiv \frac{2\pi\epsilon \cdot x_+}{4G_3} = S_{\rm BH}\,,$$

$$\ell_3 M_{\rm BTZ} = \frac{x_+^2 + x_-^2}{8\ell_3 G_3}\epsilon = \frac{2M\mathbf{W}_s - \mathbf{Y}_s(\hat{a}^2 + \hat{b}^2)}{4\ell^2\sqrt{\mathbf{V}_s}}\ N^2\epsilon$$

$$J_{\rm BTZ} = \frac{x_+ x_-}{4\ell_3 G_3}\epsilon = \frac{\hat{a}\hat{b}}{2\ell^2} N^2\epsilon$$

- EVH black hole $\xrightarrow{\text{Near Horizon}}$ Pinching AdS_3
- Near EVH black hole $\xrightarrow{\text{Near Horizon}}$ Pinching BTZ black hole
- 10d entropy is given by BTZ entropy

Rotating (near-)EVH:

- AdS_3/CFT_2: Pinching $AdS_3 \Rightarrow$ dual CFT_2
- Brown Henneaux: $c_L = c_R$ (including pinching)

$$c_{\text{rotating}} = \frac{3\ell_3}{2G_3}\epsilon = \frac{3(a^2+q)}{\ell^4 \Xi_a}\sqrt{\frac{a^2\ell^2+q^2}{\mathbf{V}}}\, N^2\epsilon$$

- Excitations:

$$L_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} - J_{\text{BTZ}}) \sim N^2\epsilon$$

$$\bar{L}_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) \sim N^2\epsilon$$

- Cardy's formula:

$$S_{\text{CFT}} = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} + 2\pi\sqrt{\frac{\bar{c}}{6}\left(\bar{L}_0 - \frac{\bar{c}}{24}\right)} = S_{BH}$$

Rotating (near-)EVH:

- AdS_3/CFT_2: Pinching $AdS_3 \Rightarrow$ dual CFT_2
- Brown Henneaux: $c_L = c_R$

$$c_{\text{rotating}} = \frac{3\ell_3}{2G_3}\epsilon = \frac{3(a^2+q)}{\ell^4 \Xi_a}\sqrt{\frac{a^2\ell^2+q^2}{\mathbf{V}}}\, N^2\epsilon$$

$$L_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} - J_{\text{BTZ}}) \sim N^2\epsilon$$

$$\bar{L}_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) \sim N^2\epsilon$$

- *finite* central charge in IR 2d CFT: *large N* limit:

$$\boxed{N^2\epsilon = \text{fixed}}$$

1. entropy $S_{\text{BH}} \sim N^2\epsilon$ finite in this limit
2. $M_{\text{BTZ}}, J_{\text{BTZ}} \sim N^2\epsilon$ also finite in this limit
3. $c_L = c_R, L_0, \bar{L}_0, S_{Cardy} \sim N^2\epsilon$ also finite in this limit

Rotating (near-)EVH:

- Brown Henneaux: $c_L = c_R$

$$c_{\text{rotating}} = \frac{3\ell_3}{2G_3}\epsilon = \frac{3(a^2+q)}{\ell^4 \Xi_a}\sqrt{\frac{a^2\ell^2+q^2}{\mathbf{V}}}\, N^2\epsilon$$

$$L_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) = \frac{\ell_3 K}{4\ell^3 \Xi_a}\left(M\mathbf{W} - a\hat{b}\sqrt{\mathbf{V}}\right) N^2\epsilon$$

$$\bar{L}_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) = \frac{\ell_3 K}{4\ell^3 \Xi_a}\left(M\mathbf{W} + a\hat{b}\sqrt{\mathbf{V}}\right) N^2\epsilon$$

- rotating EVH point $(a, 0, q; m(a, q))$ determines the IR 2d CFT central charge and vacuum structure, whereas its orthogonal deformations encode its excitations.

- Static (near-)EVH:

$$c_{\text{static}} = \frac{3\ell_3}{2G_3}\epsilon = \frac{3q^2}{\ell^4\sqrt{1+\frac{2q}{\ell^2}}}\,N^2\epsilon$$

$$L_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) \sim N^2\epsilon$$
$$\bar{L}_0 - \frac{c}{24} = \frac{1}{2}(\ell_3 M_{\text{BTZ}} + J_{\text{BTZ}}) \sim N^2\epsilon$$

- *finite* central charge and *finite* gap in IR 2d CFT: *large N* limit:

$$\boxed{N^2\epsilon = \text{fixed}}\,:$$

1. entropy $S_{\text{BH}} \sim N^2\epsilon$ finite in this limit
2. $M_{\text{BTZ}}, J_{\text{BTZ}} \sim N^2\epsilon$ also finite in this limit
3. $c_L = c_R,\, L_0,\, \bar{L}_0,\, S_{Cardy} \sim N^2\epsilon$ also finite in this limit

Static (near-)EVH:

$$c_{\text{static}} = \frac{3\ell_3}{2G_3}\epsilon = \frac{3q^2}{\ell^4\sqrt{1+\frac{2q}{\ell^2}}}N^2\epsilon$$

$$L_0 - \frac{c}{24} = \frac{1}{8\ell^2\sqrt{\mathbf{V}_s}}\left(2M\mathbf{W}_s - \mathbf{Y}_s(\hat{a}^2+\hat{b}^2) - 2\hat{a}\hat{b}\sqrt{\mathbf{V}_s}\right)N^2\epsilon$$

$$\bar{L}_0 - \frac{c}{24} = \frac{1}{8\ell^2\sqrt{\mathbf{V}_s}}\left(2M\mathbf{W}_s - \mathbf{Y}_s(\hat{a}^2+\hat{b}^2) + 2\hat{a}\hat{b}\sqrt{\mathbf{V}_s}\right)N^2\epsilon$$

- static EVH point $(0, 0, q; m(q))$ determines the IR 2d CFT by fixing its central charge
- orthogonal deformations encode *finite* excitations

- 10d dimensional black hole has dual description in terms of $\mathcal{N} = 4$ SYM on boundary of AdS_5.
- NH limit of AdS_5 black hole $\leftrightarrow$ low energy limit of dual CFT_4.
- CFT_4 dual to asymptotically AdS_5 black hole = UV CFT.
- Near Horizon limit of CFT_4 = IR CFT.
- relate quantum numbers of IR theory to those of NH CFT_2.

- UV quantum numbers of scalar field: eigenvalues of operators

$$\Delta_{\rm UV} = \ell E = i\ell\partial_t, \qquad J_{a,b} = -i\partial_{\phi s,\psi s} \qquad J_{i,3} = -i\partial_{\psi_{i,3}}.$$

- IR quantum numbers of scalar field: eigenvalues of operators

$$\Delta_{\rm IR} = i\ell_3\partial_\tau, \qquad J_{\tilde{chi}} = -i\partial_{\tilde{chi}} \qquad J_\xi = -i\partial_\xi\,.$$

IR-UV charge mapping, rotating EVH case

Charges Have a Near-EVH Expansion:

$$\boxed{Z = Z_{EVH} + \epsilon^p Z^{(p)}}, \qquad \text{where}$$

- Z_{EVH} is the value at the EVH point.
- $Z^{(p)}$ are the near-EVH excitations.

IR-UV charge mapping, rotating EVH case

Use chain rule to express IR charges in terms of UV ones.

- **In the IR limit:**

$$J_\xi = -i\left(\frac{\partial\psi}{\partial\xi}\frac{\partial}{\partial\psi} + \frac{\partial\psi_3}{\partial\xi}\frac{\partial}{\partial\psi_3}\right) = \partial_\xi\psi J_b + \partial_\xi\psi_3 J_3 \sim N^2\epsilon^2,$$

$$J_{\tilde\chi} = \partial_{\hat\chi}\psi J_b + \partial_{\hat\chi}\psi_3 J_3 = \frac{(a^2+q)}{2\ell^2\Xi_a\sqrt{a^2\ell^2+q^2}}a\hat{b}\, N^2\epsilon = J_{\mathrm{BTZ}}.$$

In the Large N limit:

- Quantum Number associated with ξ scales like $N^2\epsilon^2$. Large N limit: $J_\xi \sim \epsilon$ is subleading
- Quantum Number associated with pinching angle: $J_{\tilde\chi}$ is finite in large N limit and matches the BTZ angular

IR-UV charge mapping, rotating EVH case

IR conformal dimension Δ_{IR}

$$\Delta_{\text{IR}} \equiv i\ell_3 \frac{\partial}{\partial \tau} = \frac{\ell_3}{\ell}\frac{K}{\epsilon}\left(i\ell \frac{\partial}{\partial t} + i\ell\Omega_a^{0S}\frac{\partial}{\partial \phi} + \sum_{i=1,2} i\ell\Omega_i^0 \frac{\partial}{\partial \psi_i}\right)$$
$$= \frac{\ell_3}{\ell}\frac{K}{\epsilon}\left(\Delta - \ell\Omega_a^{0S} J_a - 2\ell\Omega_1^0 J_1\right).$$

Then conformal dimension given by function of EVH parameters + BTZ mass:$\Delta_{\text{IR}} = \Delta^0_{\text{IR}} + \ell_3 M_{\text{BTZ}}$, where $\ell M_{\text{BTZ}} = K(\Delta^{(2)} - \ell\Omega_a^0 J_a^{(2)} - 2\ell\Omega_1^0 J_1^{(2)})\epsilon$ and

$$\Delta^0_{\text{IR}} = \Delta^0_{\text{IR}}(\Delta^0, J_a^0, J_1^0).$$

Rotating Near-EVH Limit

- UV charges $\xrightarrow{\text{Near-EVH}}$ IR charges **given by CFT_2 charges.**
- Suggests that **near-EVH sector in UV 4d dual is sector described by IR 2d dual.**
- Near horizon information given by 2d CFT: **evidence for EVH/CFT_2 Correspondence.**

Static near-EVH Case

Charges Have a Near-EVH Expansion:

$$\boxed{Z = Z_{EVH} + \epsilon^p Z^{(p)}}, \qquad \text{where}$$

- Z_{EVH} is the value at the EVH point.
- $Z^{(p)}$ are the near-EVH excitations.

Quantum number associated with pinching angle: $J_{\tilde{\chi}}$ is finite in large N limit; given by BTZ angular momentum + some extra terms

$$J_{\tilde{\chi}} = -i\partial_{\tilde{\chi}} = -i\left(-\frac{1}{\epsilon}\partial_{\psi_3}\right) = -\frac{1}{\epsilon}J_3$$

$$= J_{\text{BTZ}} - \frac{\ell}{2q}(\hat{a}J_b + \hat{b}J_a).$$

IR conformal dimension Δ_{IR}

$$\Delta_{\text{IR}} \equiv i\ell_3 \frac{\partial}{\partial \tau} = \frac{\ell_3}{\ell}\frac{K}{\epsilon}\left(i\ell\frac{\partial}{\partial t} + 2i\ell\Omega_1^0 \frac{\partial}{\partial \psi_1}\right)$$
$$= \frac{\ell_3}{\ell}\frac{K}{\epsilon}\left(\Delta - 2\ell\Omega_1^0 J_1\right).$$

Then conformal dimension given by function of EVH parameters + BTZ mass + extra terms:

$$\Delta_{\text{IR}} = \Delta_{\text{IR}}^0 + \ell_3 M_{\text{BTZ}} + \frac{\ell_3\ell}{2q\sqrt{q}}\mathbf{Y}_s(\hat{a}J_b + \hat{b}J_a),$$

where

$$\Delta_{\text{IR}}^0 = \Delta_{\text{IR}}^0(\Delta^0, J_1^0).$$

and

$$\ell M_{\text{BTZ}} = K(\Delta^{(2)} - \ell\Omega_a^0 J_a^{(2)} - 2\ell\Omega_1^0 J_1^{(2)})\epsilon.$$

Static Near-EVH Limit

- IR charges rearrange into CFT_2 charges **+ extra terms.**
- Extra terms due to rotation on S^3 in NH limit.

First law of thermodynamics, IR vs. UV, 3d vs. 5d

- **10d First Law:**

$$T_{\rm H} dS_{\rm BH} = dE - 2\Omega_1 dJ_1 - \Omega_a dJ_a - \Omega_b dJ_b - \Omega_3 dJ_3$$

- For a fixed point in parameter space, physical variations belong to the subspace of orthogonal deformations to the EVH hyperplane, leaving the EVH point fixed.
- eg: $E = E^0 + \epsilon^2 E^{(2)}(\hat{b}, M)$. Then $dE = 0 + \epsilon^2 dE^{(2)}$.

Rotating near-EVH case

- LHS:

$$T_{\mathrm{H}} dS_{\mathrm{BH}} = \frac{\epsilon}{K} T_{\mathrm{BTZ}} dS_{\mathrm{BTZ}} .$$

- RHS 1;

$$\Omega_b dJ_b + \Omega_3 dJ_3 = \frac{\epsilon}{K} \Omega_{\mathrm{BTZ}} dJ_{\mathrm{BTZ}} ,$$

- **Thermodynamic quantities associated to pinching NH circle give BTZ angular momentum term (up to scaling)**
- RHS 2:

$$\left(dE - 2\Omega_1 dJ_1 - \Omega_a^R dJ_a \right) + \mathcal{O}(\epsilon^2) = \frac{\epsilon}{K} dM_{\mathrm{BTZ}}$$

- **Remaining pieces rearrange to give BTZ mass term (up to scaling)**

Rotating near-EVH case

$$T_{\rm H} dS_{\rm BH} = dE - 2\Omega_1 dJ_1 - \Omega_a dJ_a - \Omega_b dJ_b - \Omega_3 dJ_3$$
$$\Downarrow$$
$$T_{\rm BTZ} dS_{\rm BTZ} = dM_{\rm BTZ} - \Omega_{\rm BTZ} dJ_{\rm BTZ}$$

The UV 10d 1st law reduces in the near-EVH approximation to an IR 1st law for BTZ black hole.

Static near-EVH case

- LHS:

$$T_{\rm H} dS_{\rm BH} = \epsilon \frac{\sqrt{q}}{\ell} T_{\rm BTZ} dS_{\rm BTZ}.$$

- RHS 1:

$$\Omega_a dJ_a + \Omega_b dJ_b + \Omega_3 dJ_3 = \frac{\sqrt{q}\epsilon}{\ell} \left(\Omega_{\rm BTZ} dJ_{\rm BTZ} + \frac{\ell \mathbf{Y}_s}{2q^{3/2}} d\left(aJ_a + bJ_b\right) \right).$$

Thermodynamic quantities associated to pinching NH circle and S^3 rotation give BTZ angular momentum term (up to scaling and extra piece)

- RHS 2:

$$dE - \sum_{i=1,2} \Omega_i^0 dJ_i = \frac{\sqrt{q}\epsilon}{\ell} \left(dM_{\rm BTZ} + \frac{\ell \mathbf{Y}_s}{2q^{3/2}} d\left(aJ_a + bJ_b\right) \right).$$

- **Remaining pieces rearrange to give BTZ mass term (up to scaling and extra piece)**

Static EVH case

$$
\begin{aligned}
T_{\rm H} dS_{\rm BH} &= dE - 2\Omega_1 dJ_1 - \Omega_a dJ_a - \Omega_b dJ_b - \Omega_3 dJ_3 \\
&\Downarrow \\
T_{\rm BTZ} dS_{\rm BTZ} &= dM_{\rm BTZ} - \Omega_{\rm BTZ} dJ_{\rm BTZ}
\end{aligned}
$$

The UV 10d 1st law reduces in the near-EVH approximation to an IR 1st law for BTZ black hole.

Relation between EVH/CFT and Kerr/CFT

- EVH/CFT correspondence: **gravity theory in NH limit of EVH black holes governed by 2d CFT.**
- Consistency check: connection between 2d CFTs in the EVH/CFT correspondence and 2d chiral CFTs in the Kerr/CFT correspondence.

Review of Kerr/CFT for AdS_5 black holes

- Near horizon geometry for **finite horizon** 5d extremal black holes embedded into 10d:

$$ds_{10}^2 = \tilde{A}(\theta_n)\left(-y^2 d\tau^2 + \frac{dy^2}{y^2}\right) + \tilde{B}_1(\theta_n){e_\phi}^2 + \tilde{B}_2(\theta_n)\left(e_\psi + C(\theta_0)^2 e_\phi\right.$$
$$+ \sum_{n,m=0}^{2} F_{\theta_n\theta_m}(\theta_n) d\theta_n d\theta_n + \sum_{i=1}^{3} D_i(\theta_n)\left(e_{\psi_i} + P_i(\theta_0)(e_\phi + e_\psi)\right)^2 ,$$

- This metric can be viewed as a warped $S^3 \times S^5$ bundle over AdS_2

Review of Kerr/CFT for AdS_5 black holes

- Each U(1): Virasoro algebra with central charge:

$$\boxed{c_\zeta = \frac{6k_\zeta S_{BH}}{\pi}}$$

- k_ζ comes from $e_\zeta = d\zeta + k_\zeta d\tau$.
- 5 central charges $\Rightarrow$ 5 dual CFT descriptions. Temperature of dual CFT:

$$\boxed{T_i = -\frac{\partial T_{\rm H}/\partial r_+}{\partial\Omega_i/\partial r_+}\bigg|_{r_+=r_0} .}$$

- Each CFT satisfies Cardy formula:

$$\boxed{S = \frac{\pi^2}{3} c_\zeta T_\zeta}$$

Taking the near-EVH limit:

- Kerr/CFT works for extremal finite size black holes
- EVH/CFT works for near-EVH black holes which are not strictly extremal
- compare proposals in region of parameter space where both apply
- restrict to extremal excitations in the EVH/CFT correspondence
- consider *vanishing horizon limit* in the Kerr/CFT correspondence

Rotating near-EVH case.

- leading terms in the Kerr/CFT central charges in the near-EVH limit:

$$c_\phi = \frac{3\hat{b}}{\ell}\frac{q + a^2\mathbf{V}^{-1}}{\ell^2\sqrt{\mathbf{V}}}N^2\epsilon^2\,, \qquad c_{\psi_1} = c_{\psi_2} = \frac{3\sqrt{q}}{\ell}\frac{a\hat{b}}{\ell^2\mathbf{V}}\frac{\ell_3}{\ell}\sqrt{\mathbf{Y}_s}N^2\epsilon^2\,,$$

$$c_\xi = \omega_3(c_\psi + (al/q)c_{\psi_3}) = 0$$

$$c_{\tilde{\chi}} = \epsilon\left(\omega_1 c_\psi + \omega_2 c_{\psi_3}\right) = \frac{3\sqrt{\mathbf{V}}}{\ell^2\Xi_a}\frac{\ell_3^2}{\ell^2}\frac{\sqrt{q^2 + a^2\ell^2}}{\ell^2}\,N^2\epsilon$$

- large N limit: $c_\phi,\ c_{\psi_1},\ c_{\psi_2};\ c_\xi \sim \epsilon \to 0$; corresponding CFTs break down.

$$\boxed{c_{\tilde{\chi}} = c_{Brown-Henneaux}}$$

- Central charge associated with AdS_3 angle $c_{\tilde{\chi}}$ exactly equals the Brown-Henneaux central charge.
- connecting Kerr/CFT and EVH/CFT: **chiral 2d CFT in Kerr/CFT is the chiral sector of CFT in the EVH/CFT**

Static near-EVH regime.

- leading terms in the Kerr/CFT central charges in the EVH limit:

$$c_\phi = \frac{3q}{\ell^2\sqrt{\mathbf{V}_s}}\frac{\hat{b}}{\ell}N^2\epsilon\,, \qquad c_\psi = \frac{3q}{\ell^2\sqrt{\mathbf{V}_s}}\frac{\hat{a}}{\ell}N^2\epsilon$$

$$c_{\psi_1} = c_{\psi_2} = \frac{6\sqrt{q}\ell_3^3}{\ell^4}\frac{\hat{a}\hat{b}}{\ell^2}\sqrt{\mathbf{Y}_s}N^2\epsilon^2\,, \qquad c_{\psi_3} = -\frac{3q^2}{\ell^4\sqrt{\mathbf{V}_s}}N^2\,,$$

- Central charge associated with AdS_3 angle $c_{\tilde{\chi}}$ exactly equals the Brown-Henneaux central charge.

$$\boxed{c_{\hat{\chi}} = -\epsilon c_{\psi_3} = c_{\text{static}}}\,.$$

- $$\boxed{c_{\text{EVH AdS}_3}\Big|_{\text{extremal}} = c_{\text{Kerr/CFT}}\Big|_{\text{near-EVH}}}\,.$$
- BUT: c_ϕ, c_ψ finite in large N limit; rotation in NH S^3...CFT??

the Kerr/CFT central charge associated with the vanishing $U(1)$ isometry cycle remains finite in the EVH limit and always matches the standard AdS_3 Brown-Henneaux central charge computed in the EVH/CFT correspondence.

Discussion

- Studied EVH, near-EVH limit of rotating 5d black holes in 10d IIB supergravity. EVH black holes: $A_H, T_H = 0$.
- EVH black hole=point on EVH surface. Near horizon geometry develops pinching AdS_3 throat $\Rightarrow$ dual CFT_2 description.
- EVH/CFT correspondence: gravity theory in NH limit of EVH black holes governed by 2d CFT.
- Othogonally displace configuration from EVH surface $\Rightarrow$ excite pinching AdS_3 to pinching BTZ.
- S_{BTZ} gives S_{10d}.
- AdS_3/CFT_2: CFT central charge and excitations.
- Combine pinching and large N limit: all BTZ and CFT_2 charges are finite.

Discussion

- Scalar probe in black hole background: map UV quantum numbers to IR ones.
- Precise mapping: View IR CFT_2 as sector in UV CFT_4.
- First law of thermodynamics for 10d $\xrightarrow{\text{near-EVH limit}}$ first law of thermodynamics of NH BTZ black hole.
- Future work: generalise this statement.
- Check of EVH/CFT proposal:

$$\boxed{c_{\text{EVH AdS}_3}\Big|_{\text{extremal}} = c_{\text{Kerr/CFT}}\Big|_{\text{near-EVH}}}:$$

  chiral CFT Kerr/CFT proposal=chiral limit of the CFT_2 in EVH/CFT correspondence.
- Role of extra finite Kerr/CFT central charges?